Plex
MAA4402 **Quizzes P** Monday 30Aug2021
MAA5404

Number Sets. Expression $k \in \mathbb{N}$ [read as "k is an element of $\mathbb{N}$" or "k in $\mathbb{N}$"] means that k is a natural number; a ***natnum***. Expression $\mathbb{N} \ni k$ [read as "$\mathbb{N}$ ***owns*** k"] is a synonym for $k \in \mathbb{N}$.

$\mathbb{N}$ = natural numbers = $\{0, 1, 2, \dots\}$.

$\mathbb{Z}$ = integers = $\{\dots, -2, -1, 0, 1, \dots\}$. For the set $\{1, 2, 3, \dots\}$ of positive integers, the ***posints***, use $\mathbb{Z}_+$. Use $\mathbb{Z}_-$ for the negative integers, the ***negints***.

$\mathbb{Q}$ = rational numbers = $\{\frac{p}{q} \mid p \in \mathbb{Z} \text{ and } q \in \mathbb{Z}_+\}$. Use $\mathbb{Q}_+$ for the positive rationals and $\mathbb{Q}_-$ for the negative rationals.

$\mathbb{R}$ = reals. The ***posreals*** $\mathbb{R}_+$ and the ***negreals*** $\mathbb{R}_-$.

$\mathbb{C}$ = complex numbers, also called the ***complexes***.

For $\omega \in \mathbb{C}$, let "$\omega > 5$" mean "ω *is* ***real*** and $\omega > 5$". [Use the same convention for $\geq, <, \leq$, and also if 5 is replaced by any real number.]

Use $\overline{\mathbb{R}} = [-\infty, +\infty] := \{-\infty\} \cup \mathbb{R} \cup \{+\infty\}$, the ***extended reals***.

An "***interval of integers***" $[b\,..\,c)$ means the intersection $[b, c) \cap \mathbb{Z}$; ditto for open and closed intervals. So $[\mathrm{e}\,..\,2\pi] = \{3, 4, 5, 6\} = [3\,..\,6] = (2\,..\,6]$. We allow b and c to be $\pm\infty$; so $(-\infty\,..\,-1]$ is $\mathbb{Z}_-$. And $[-\infty\,..\,-1]$, is $\{-\infty\} \cup \mathbb{Z}_-$.

Floor function: $\lfloor \pi \rfloor = 3$, $\lfloor -\pi \rfloor = -4$. Ceiling fnc: $\lceil \pi \rceil = 4$. Absolute value: $|-6| = 6 = |6|$ and $|-5 + 2\mathbf{i}| = \sqrt{29}$.

Mathematical objects. Seq: '*sequence*'. poly(s): '*polynomial(s)*'. irred: '*irreducible*'. Coeff: '*coefficient*' and var(s): '*variable(s)*' and parm(s): '*parameter(s)*'. Expr.: '*expression*'. Fnc: '*function*' (so ratfnc: means rational function, a ratio of polynomials). trnfn: '*transformation*'. cty: '*continuity*'. cts: '*continuous*'. diff'able: '*differentiable*'. CoV: '*Change-of-Variable*'. CoI: '*Constant of Integration*'. LoI: '*Limit(s) of Integration*'. RoC: '*Radius of Convergence*'.

Soln: '*Solution*'. Thm: '*Theorem*'. Prop'n: '*Proposition*'. CEX: '*Counterexample*'. eqn: '*equation*'. RhS: '*RightHand side*' of an eqn or inequality. LhS: '*lefthand side*'. Sqrt or Sqroot: '*square-root*', e.g, "the sqroot of 16 is 4". Ptn: '*partition*', but pt: '*point*' as in "a fixed-pt of a map".

Binop: '*Binary operator*'. Binrel: '*Binary relation*'.

FTC: '*Fund. Thm of Calculus*'. IVT: '*intermediate-Value Thm*'. MVT: '*Mean-Value Thm*'.

The ***logarithm*** function, defined for $x>0$, is $\log(x) := \int_1^x \frac{\mathrm{d}v}{v}$. Its inverse-fnc is $\exp()$.

For $x>0$, then, $\exp(\log(x)) = x = \mathrm{e}^{\log(x)}$. For real t, naturally, $\log(\exp(t)) = t = \log(\mathrm{e}^t)$.

PolyExp: '*Polynomial-times-exponential*', e.g, $[3 + t^2]\cdot\mathrm{e}^{4t}$. PolyExp-sum: '*Sum of polyexps*'. E.g, $f(t) := 3t\mathrm{e}^{2t} + [t^2]\cdot\mathrm{e}^t$ is a polyexp-sum.

Phrases. WLOG: '*Without loss of generality*'. IFF: '*if and only if*'. TFAE: '*The following are equivalent*'. ITOf: '*In Terms Of*'. OTForm: '*of the form*'. FTSOC: '*For the sake of contradiction*'. And ※="Contradiction".

IST: '*It Suffices To*', as in ISTShow, ISTExhibit.

Use w.r.t: '*with respect to*' and s.t: '*such that*'.

Latin: e.g: *exempli gratia*, '*for example*'. i.e: *id est*, '*that is*'. N.B: *Nota bene*, '*Note well*'. inter alia: '*among other things*'. QED: *quod erat demonstrandum*, meaning "end of proof".

Sequence notation. A *sequence* $\vec{\mathbf{x}}$ abbreviates $(x_0, x_1, x_2, x_3, \dots)$. For a set Ω, expression "$\vec{\mathbf{x}} \subset \Omega$" means $[\forall n\colon x_n \in \Omega]$. Use $\mathrm{Tail}_N(\vec{\mathbf{x}})$ for the subsequence

$$(x_N, x_{N+1}, x_{N+2}, \dots)$$

of $\vec{\mathbf{x}}$. Given a fnc $f\colon\Omega\to\Lambda$ and an Ω-sequence $\vec{\mathbf{x}}$, let $f(\vec{\mathbf{x}})$ be the Λ-sequence $(f(x_1), f(x_2), f(x_2), \dots)$.

Suppose Ω has an addition and multiplication. For Ω-seqs $\vec{\mathbf{x}}$ and $\vec{\mathbf{y}}$, then, let $\vec{\mathbf{x}} + \vec{\mathbf{y}}$ be the sequence whose n^{th} member is $x_n + y_n$. I.e

$$\vec{\mathbf{x}} + \vec{\mathbf{y}} = [n \mapsto [x_n + y_n]].$$

Similarly, $\vec{\mathbf{x}} \cdot \vec{\mathbf{y}}$ denotes seq $[n \mapsto [x_n \cdot y_n]]$.

Plex notation. Let *SCC* mean "positively oriented simple-closed-contour". For a SCC C, have $\mathring{\mathsf{C}}$ be the (open) region C encloses, and let $\widehat{\mathsf{C}}$ mean C together with $\mathring{\mathsf{C}}$. So $\widehat{\mathsf{C}}$ is $\mathsf{C} \sqcup \mathring{\mathsf{C}}$; it is automatically simply-connected and is a closed bounded set.

Plex [2021t] quizzes so far...

P1: Wed. 01 Sep Blanks$\in\mathbb{R}$. So $\frac{1}{2+3\mathbf{i}} =$ ________ $+\mathbf{i}\cdot$ [________].

Thus $\dfrac{5-\mathbf{i}}{2+3\mathbf{i}} =$ ________________ $+\mathbf{i}\cdot$ [________________].

P2: Mon. 11 Oct With $\alpha := [3\mathbf{i}]^3$ and $x + \mathbf{i}y := \operatorname{Log}(\alpha)$, then $x=$ ____________________ *and* $y=$ ____________________.

P3: Wed. 13 Oct [Below, "Log" is the P.V. of $\mathbb{C}$-logarithm, and $z,w \in \mathbb{C}$.]

If $\operatorname{Re}(z)<0$ then $\operatorname{Log}(z^2) = 2\operatorname{Log}(z)$. *AT AF Nei*

If $\operatorname{Re}(z)>1$ then $\operatorname{Log}(z^3) = 3\operatorname{Log}(z)$. *AT AF Nei*

If $\operatorname{Re}(z)=0$ and $\operatorname{Im}(z)>2$ then $\operatorname{Log}(\exp(z)) = z$. *AT AF Nei*

If $\operatorname{Re}(z)>0$ and $\operatorname{Re}(w)>0$ then $\operatorname{Log}(z\cdot w) = \operatorname{Log}(z) + \operatorname{Log}(w)$. *AT AF Nei*

So $\operatorname{Log}([1+\mathbf{i}]^{13}) = 13\ln(\sqrt{2}) + y\mathbf{i}$, where $y=$ ________.

P4: Fri. 15 Oct With L the line-segment contour from $\mathbf{p} := 4$ to $\mathbf{q} := 7+\mathbf{i}$, compute $J := \int_{\mathsf{L}} \overline{z}\, \mathrm{d}z$, showing the parametrization you used and resulting definite integral.

P5: Fri. 22 Oct Let $\mathsf{C} := \mathrm{Sph}_3(\mathbf{i})$, a radius 3 circle centered at $\mathbf{i}$. Integral

$$\oint_{\mathsf{C}} \frac{\cos(z^2)}{1-3z}\, \mathrm{d}z = \text{.................................}.$$

[*Hint:* Rewrite, then use the Cauchy Integral Formula.]

P6: Mon. 25 Oct Let R be the pos-oriented rectangle with corners $2 \pm \mathbf{i}$ and $-9 \pm \mathbf{i}$.

Then $\oint_{\mathsf{R}} \frac{\exp(2z)}{z^2 - 9}\, dz =$ ⌞..........................⌟.

Let $\mathsf{C} := \operatorname{Sph}_3(\mathbf{i})$, a radius 3 circle centered at $\mathbf{i}$. Integral $\oint_{\mathsf{C}} \frac{\cos(z^2)}{3 - z}\, dz =$ ⌞..................................⌟.

P: Wed. 27 Oct *Class-W, in-class exam. Bring lots of paper, and perhaps colored pencils for drawing pictures.*

P7: Fri. 29 Oct Let $f(x+\mathbf{i}y) := 4x^3 + \mathbf{i}y^3$. Let $\mathbf{D},\mathbf{H},\mathbf{C} \subset \mathbb{C}$ be the sets where f is, respectively: **D***ifferentiable*, **H***olomorphic*, *has* **C***ontinuous 1*st*-partials*. Then

D = ...;

H = ...;

C = ...

[You may use: *Set-builder notation. List pts between braces. Union & intersection symbols. Sets* $\mathbb{C}$, $\mathbb{R}$, $\varnothing$.]

P8: Fri. 05 Nov Compute $J := \oint_{\mathsf{C}} \frac{\cos(2z)}{[z-4][z-100]}\,\mathrm{d}z$, where $\mathsf{C} := \mathrm{Sph}_9(\mathbf{i})$, the radius-9 circle centered at $\mathbf{i}$.

PBonus: Mon. 08 Nov *Our W-Bonus was due at the Beginning of Class, hopeully nicely typeset and well-written.*

P9: Wed. 10 Nov Prof. K thinks the flu vaccine is *Good Idea*, <u>even</u> if PUBLIX offers no gift-card. Circle: Yes True

The visual representation of $\mathbb{C}$ is sometimes called "the ? plane", where ? is Circle: **Unreal** **Higher** **Snakes-on-a** **Argand** **Krypton** **Rayon** **Xenon** **Euler** **Goursat** **Liouville** **No-need-to-**x y**-com** **Air** **Sea** **De** **Rain-in-Spain-stays-mainly-on-the** .

PA: Fri. 12 Nov Let $f(z) := z^4 \exp(2/z)$.
Then $\operatorname{Res}(f, 0) =$ ....

[Hint: *Write the PS for* e^w, *then plug in* $2/z$ *for* w. *Multiply the resulting Laurent Series by* z^4. *You may use the factorial symbol in expressing your answer.*]

For a LOR (letter-of-recommendation), Prof. K requires two courses, or a Special Topics or graduate course [Circle]:

Yes **True** **Darn tootin'!**

PX: Wed. 17 Nov *Class-X*, our third in-class exam.

PB: Fri. 19 Nov Let $f(z) := z^4 + 13z^2 + 36$. Reciprocal $H(z) := 1/f(z)$ has, in the upper half-plane, two poles $\mathbf{p}$ and $\mathbf{q}$, where $\mathbf{p}$ lies closer to the origin than $\mathbf{q}$.

So $\operatorname{Res}(H, \mathbf{p}) =$ .... *and* $\operatorname{Res}(H, \mathbf{q}) =$ ....

Our D-contour technique applies to H.

Thus $J := \int_{-\infty}^{+\infty} \frac{1}{z^4 + 13z^2 + 36}\, dx =$ ....

PC: Mon. 22 Nov STMT: Most folks like Tnanksgiving Break. (Circle)

T *F* *What? We don't have class?*

A function holomorphic on open $\operatorname{Bal}_2(0)$ *with poles at* $\pm 2\mathbf{i}$ *and at* ± 2 *is* $h(z) :=$ ....

Prof. King believes that writing in complete, coherent sentences is crucial in communicating Mathematics, improves posture, and whitens teeth. Circle one:

True! **Yes!** ***wH'at S a?sEnTENcE***

PThanksgiving: Wed-Sat. 24-27 Nov *No classes, so . . . Lots of time to post solutions to our Archive!*

PD: Mon. 29 Nov The IOP (Individual Optional Project) must be carefully TYPESET. It is due by 2PM on Thursday, 07Dec2023, slid *completely* under my office door, Little Hall 402 (northeast corner of top floor) [Circle]: Yes Cool! Thanks

---

On annulus $\{2 < |z| < \infty\}$, fnc $f(z) := 1/[z - 2\mathbf{i}]$ has Laurent series $\sum_{n=-\infty}^{\infty} B_n z^n$, where $B_{-4} =$ ________, $B_{-3} =$ ________, and $B_2 =$ ________.

PE: Wed. 01 Dec Residue $\operatorname{Res}_{z=0}\left(z^6 \cdot \sin\left(\frac{1}{3z}\right)\right) =$ ________________.

PF: Wed. 08 Dec Prof. King thinks that submitting a ROBERT LONG PRIZE ESSAY [typically 2 prizes, \$600 total] is a *really good idea.* A ten-page essay is fine. Date for the emailed-PDF is Mon., 25 Mar., 2024.

[Circle]: **Yes** **True** ***Résumé material!***