Plex
MAA4402 **Quizzes P** Tuesday 03Jan2017
MAA5404

Number Sets. Expression $k \in \mathbb{N}$ [read as "k is an element of $\mathbb{N}$" or "k in $\mathbb{N}$"] means that k is a natural number; a ***natnum***. Expression $\mathbb{N} \ni k$ [read as "$\mathbb{N}$ ***owns*** k"] is a synonym for $k \in \mathbb{N}$.

$\mathbb{N}$ = natural numbers = $\{0, 1, 2, \ldots\}$.

$\mathbb{Z}$ = integers = $\{\ldots, -2, -1, 0, 1, \ldots\}$. For the set $\{1, 2, 3, \ldots\}$ of positive integers, the ***posints***, use $\mathbb{Z}_+$. Use $\mathbb{Z}_-$ for the negative integers, the ***negints***.

$\mathbb{Q}$ = rational numbers = $\{\frac{p}{q} \mid p \in \mathbb{Z} \text{ and } q \in \mathbb{Z}_+\}$. Use $\mathbb{Q}_+$ for the positive rationals and $\mathbb{Q}_-$ for the negative rationals.

$\mathbb{R}$ = reals. The ***posreals*** $\mathbb{R}_+$ and the ***negreals*** $\mathbb{R}_-$.

$\mathbb{C}$ = complex numbers, also called the ***complexes***.

For $\omega \in \mathbb{C}$, let "$\omega > 5$" mean "ω *is* ***real*** *and* $\omega > 5$". [Use the same convention for $\geq, <, \leq$, and also if 5 is replaced by any real number.]

Use $\overline{\mathbb{R}} = [-\infty, +\infty] := \{-\infty\} \cup \mathbb{R} \cup \{+\infty\}$, the ***extended reals***.

An "***interval of integers***" $[b\,..\,c)$ means the intersection $[b, c) \cap \mathbb{Z}$; ditto for open and closed intervals. So $[\mathrm{e}\,..\,2\pi] = \{3, 4, 5, 6\} = [3\,..\,6] = (2\,..\,6]$. We allow b and c to be $\pm\infty$; so $(-\infty\,..\,-1]$ is $\mathbb{Z}_-$. And $[-\infty\,..\,-1]$, is $\{-\infty\} \cup \mathbb{Z}_-$.

Floor function: $\lfloor \pi \rfloor = 3$, $\lfloor -\pi \rfloor = -4$. Ceiling fnc: $\lceil \pi \rceil = 4$. Absolute value: $|-6| = 6 = |6|$ and $|-5 + 2\mathbf{i}| = \sqrt{29}$.

Mathematical objects. Seq: '*sequence*'. poly(s): '*polynomial(s)*'. irred: '*irreducible*'. Coeff: '*coefficient*' and var(s): '*variable(s)*' and parm(s): '*parameter(s)*'. Expr.: '*expression*'. Fnc: '*function*' (so ratfnc: means rational function, a ratio of polynomials). trnfn: '*transformation*'. cty: '*continuity*'. cts: '*continuous*'. diff'able: '*differentiable*'. CoV: '*Change-of-Variable*'. CoI: '*Constant of Integration*'. LoI: '*Limit(s) of Integration*'. RoC: '*Radius of Convergence*'.

Soln: '*Solution*'. Thm: '*Theorem*'. Prop'n: '*Proposition*'. CEX: '*Counterexample*'. eqn: '*equation*'. RhS: '*RightHand side*' of an eqn or inequality. LhS: '*lefthand side*'. Sqrt or Sqroot: '*square-root*', e.g, "the sqroot of 16 is 4". Ptn: '*partition*', *but* pt: '*point*' as in "a fixed-pt of a map".

Binop: '*Binary operator*'. Binrel: '*Binary relation*'.

FTC: '*Fund. Thm of Calculus*'. IVT: '*intermediate-Value Thm*'. MVT: '*Mean-Value Thm*'.

The ***logarithm*** function, defined for $x>0$, is $\log(x) := \int_1^x \frac{\mathrm{d}v}{v}$. Its inverse-fnc is $\exp()$.

For $x>0$, then, $\exp(\log(x)) = x = \mathrm{e}^{\log(x)}$. For real t, naturally, $\log(\exp(t)) = t = \log(\mathrm{e}^t)$.

PolyExp: '*Polynomial-times-exponential*', e.g, $[3 + t^2]\cdot\mathrm{e}^{4t}$. PolyExp-sum: '*Sum of polyexps*'. E.g, $f(t) := 3t\mathrm{e}^{2t} + [t^2]\cdot\mathrm{e}^t$ is a polyexp-sum.

Phrases. WLOG: '*Without loss of generality*'. IFF: '*if and only if*'. TFAE: '*The following are equivalent*'. ITOf: '*In Terms Of*'. OTForm: '*of the form*'. FTSOC: '*For the sake of contradiction*'. And ※ ="Contradiction".

IST: '*It Suffices To*', as in ISTShow, ISTExhibit.

Use w.r.t: '*with respect to*' and s.t: '*such that*'.

Latin: e.g: *exempli gratia*, '*for example*'. i.e: *id est*, '*that is*'. N.B: *Nota bene*, '*Note well*'. inter alia: '*among other things*'. QED: *quod erat demonstrandum*, meaning "end of proof".

Plex notation. Let ***SCC*** mean "positively oriented simple-closed-contour". For a SCC C, have $\mathring{\mathrm{C}}$ be the (open) region C encloses, and let $\widehat{\mathrm{C}}$ mean C together with $\mathring{\mathrm{C}}$. So $\widehat{\mathrm{C}}$ is $\mathrm{C} \sqcup \mathring{\mathrm{C}}$; it is automatically simply-connected and is a closed bounded set.

Plex [2017g] quizzes so far...

P1: Wed. 11 Jan `complex-recip.tex` Blanks∈ℝ. So $\frac{1}{2+3\mathbf{i}} =$ ⌞ ⌟ $+\mathbf{i}\cdot$[⌞ ⌟].

Thus $\frac{5 - \mathbf{i}}{2 + 3\mathbf{i}} =$ ⌞................⌟ $+\mathbf{i}\cdot$[⌞................⌟].

`c.Im-of-z.tex` For z complex, $\mathrm{Im}(z) = \mathrm{Formula}(z, \overline{z}) =$ ⌞ ⌟.

P2: Wed. 18 Jan `complex-sqroot2.tex` Suppose $[C + D\mathbf{i}]^2 = -4\mathbf{i}$, where C,$D \in \mathbb{R}$. Then

$C =$ ⌞..................⌟ and $D =$ ⌞..................⌟.

`MS-defn-hints.tex` On a set Ω, a ***metric*** is a map $d\colon \Omega \to [0, \infty)$ such that $\forall w, x, y, z \in \Omega$:

MS1: ____.

MS2: ____.

MS3: ____.

P3: Fri. 03 Feb `complex-recip.tex` Blanks$\in\mathbb{R}$. So $\frac{1}{4-3\mathbf{i}} = \_\_ + \mathbf{i}\cdot\left[\_\_\right]$.

P4: Fri. 17 Feb `laplacian.tex` Fnc $u(x,y) := \cos(y\cdot x) - 7x$ maps $\mathbb{R}\times\mathbb{R}\to\mathbb{R}$. Its Laplacian is $[\Delta(u)](x,y) =$ ________________________________.

There exists function $v{:}\,\mathbb{R}\times\mathbb{R}\to\mathbb{R}$ such that $f(x+\mathbf{i}y) := u(x,y) + \mathbf{i}v(x,y)$ is holomorphic. *T* *F*

PBonus: Fri. 03 Mar `complex.int-contour-deriv2.tex` Let C be SCC $\text{Sph}_7(0)$, a circle of radius 7. Then

$$\oint_C \frac{\cos(2z)}{[z-5]^4}\,dz = \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_}$$

[Ans may be written as a product, using powers and factorials.]

P5: Mon. 13 Mar `complex.PV-exp.tex` Writing $x + \mathbf{i}y :=$ [P.V of $\mathbf{i}^{[1+\mathbf{i}]}$] with $x,y\in\mathbb{R}$, then $x =$ _____ and $y =$ _____.

P6: Fri. 14 Apr `complex.Rouche0.miss-hypoth.tex` For a SCC C, suppose fncs analytic on $\widehat{\mathsf{C}}$ satisfy that $|f(z)| \geq |g(z)|$ for every $z\in\mathsf{C}$. If $f+g$ has fewer zeros in $\mathring{\mathsf{C}}$ than f does, then there must exist a point $w\in\mathsf{C}$ such that ________.

P7: Mon. 17 Apr `complex.Rouche2.tex` Let $f(z) := z^5 + 3z^4 + 6$, and $\mathsf{C}_r := \text{Sph}_r(0)$. Our f

has ________ zeros inside C_1, and ________ zeros inside C_2.

Games Party!