Number Sets. An expression such as $k \in \mathbb{N}$ (read as "k is an element of $\mathbb{N}$" or "k in $\mathbb{N}$") means that k is a natural number; a ***natnum***.

$\mathbb{N}$ = natural numbers = $\{0, 1, 2, \ldots\}$.

$\mathbb{Z}$ = integers = $\{\ldots, -2, -1, 0, 1, \ldots\}$. For the set $\{1, 2, 3, \ldots\}$ of positive integers, the ***posints***, use $\mathbb{Z}_+$. Use $\mathbb{Z}_-$ for the negative integers, the ***negints***.

$\mathbb{Q}$ = rational numbers = $\{\frac{p}{q} \mid p \in \mathbb{Z} \text{ and } q \in \mathbb{Z}_+\}$. Use $\mathbb{Q}_+$ for the positive ***ratnums*** and $\mathbb{Q}_-$ for the negative ratnums.

$\mathbb{R}$ = reals. The ***posreals*** $\mathbb{R}_+$ and the ***negreals*** $\mathbb{R}_-$.

$\mathbb{C}$ = complex numbers, also called the ***complexes***.

An "***interval of integers***" $[b \,..\, c)$ means the intersection $[b, c) \cap \mathbb{Z}$; ditto for open and closed intervals. So $[e \,..\, 2\pi] = \{3, 4, 5, 6\} = [3 \,..\, 6] = (2 \,..\, 6]$. We allow b and c to be $\pm\infty$; so $(-\infty \,..\, -1]$ is $\mathbb{Z}_-$.

Floor function: $\lfloor \pi \rfloor = 3$ and $\lfloor -\pi \rfloor = -4$. Ceiling: $\lceil \pi \rceil = 4$. Abs.value: $|-6| = 6 = |6|$ and $|-5 + 2i| = \sqrt{29}$.

Mathematical objects. Seq: '*sequence*'. poly(s): '*polynomial(s)*'. irred: '*irreducible*'. Coeff: '*coefficient*' and var(s): '*variable(s)*' and parm(s): '*parameter(s)*'. Expr.: '*expression*'. CoI: '*Constant of Integration*'. LoI: '*Limit(s) of Integration*'.

Fnc: '*function*' (so ratfnc: means rational function, a ratio of polynomials). cty: '*continuity*'. cts: '*continuous*'. diff'able: '*differentiable*'.

Soln: '*Solution*'. Thm: '*Theorem*'. Prop'n: '*Proposition*'. CEX: '*Counterexample*'. eqn: '*equation*'. RhS: '*RightHand Side*' of an eqn or inequality. LhS: '*lefthand side*'. Sqrt or Sqroot: '*square-root*', e.g, "the sqroot of 16 is 4". Ptn: '*partition*', *but* pt: '*point*', as in "a fixed-pt of a map".

FTC: '*Fund. Thm of Calculus*'. IVT: '*intermediate-Value Thm*'. MVT: '*Mean-Value Thm*'. CoV: '*Change-of-Variable*'.

Phrases. WLOG: '*Without loss of generality*'. TFAE: '*The following are equivalent*'. ITOf: '*In Terms Of*'. OTForm: '*of the form*'. FTSOC: '*For the sake of contradiction*'. Use iff: '*if and only if*'.

IST: '*It Suffices to*' as in ISTShow, ISTExhibit.

Use w.r.t: '*with respect to*' and s.t: '*such that*'.

Latin: e.g: *exempli gratia*, '*for example*'. i.e: *id est*, '*that is*'. QED: *quod erat demonstrandum*, meaning "end of proof".

LinA [2016g] quizzes so far...

Q1: Wed. 20 Jan Consider $h\colon [1, 10] \to \mathbb{R}$ by $h(x) := 2\sin(x)$. Then

Supp(h) = ______________________________ .

Q2: Tue. 26 Jan A system of 3 linear equations in unknowns $x_1, \ldots, x_5$ reduces to the augmented matrix

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 1 & | & 12 \\ 0 & 0 & 1 & 0 & -8 & | & 34 \\ 0 & 0 & 0 & 1 & 5 & | & -56 \end{bmatrix},$$

which is in RREF. Please circle each *pivot entry.*

OYOP, describe the *general solution* in this form,

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} ? \\ ? \\ ? \\ ? \\ ? \end{bmatrix} + \alpha \begin{bmatrix} ? \\ ? \\ ? \\ ? \\ ? \end{bmatrix} + \beta \begin{bmatrix} ? \\ ? \\ ? \\ ? \\ ? \end{bmatrix} + \gamma \begin{bmatrix} ? \\ ? \\ ? \\ ? \\ ? \end{bmatrix} + \ldots$$

where each $\alpha, \beta, \gamma, \delta, \ldots$ is a free variable (either x_1 or...or x_5), and each column vector has specific numbers in it. Dim(*SolnFlat*) = ______________________ .

Q3: Wed. 27 Jan Let $B := \begin{bmatrix} 1 & 2 & 1 & 0 & 1 \\ 3 & 6 & 0 & -3 & 0 \\ 2 & 4 & -1 & -3 & -1 \\ 4 & 8 & 1 & -3 & 0 \end{bmatrix}$. Then $R :=$ *RREF*(B) is [show no work, here]

R =

.

Q4: Mon. 01 Feb $M := \begin{bmatrix} 70 & 7 \\ 1 & 2 \end{bmatrix}$. Compute M^{-1} over these three fields. [Write your $\mathbb{Z}_p$ answers using symmetric residues.]

Over $\mathbb{Z}_5$: M^{-1}= __________ . Over $\mathbb{Z}_7$: M^{-1}= __________ .

Over $\mathbb{Q}$: M^{-1}= ______________________________ .

Q5: Wed. 10 Feb Suppose $\{\mathbf{v}, \mathbf{w}, \mathbf{x}\}$ is a basis of $\mathbf{V} := \mathbb{R}^3$, a $\mathbb{R}$-VS.

Then $\{\mathbf{v}+\mathbf{x}, \mathbf{w}, \mathbf{x}\}$ *is a* $\mathbf{V}$*-basis.* *AT AF Nei*

Then $\{2\mathbf{v}, 3\mathbf{w}, 4\mathbf{x}\}$ *is a* $\mathbf{V}$*-basis.* *AT AF Nei*

Q6: Tue. 16 Feb *We discussed describing* Det *in terms of permutations, and defined the sign of a permutation. I then put the following two permutations on the board, pictorially, as non-attacking rook placements.*

Perm $\pi := [2,3,5,4,1]$ has $\text{Sgn}(\pi) =$ **+1** **−1**.

Perm $\nu := [5,2,6,4,1,3]$ has $\text{Sgn}(\nu) =$ **+1** **−1**.

Q7: Fri. 26 Feb Determinant of $\mathsf{M} := \begin{bmatrix} 3 & 4 \\ 7 & 5 \end{bmatrix}$ is ________. The characteristic-poly of M is $\wp_{\mathsf{M}}(x) = x^2 + Bx + C$, where $B=$ ________ and $C=$ ________.

Q8: Mon. 07 Mar Matrix $\mathsf{M} := \begin{bmatrix} 2 & 0 & 1 \\ 5 & 0 & -3 \\ 6 & 7 & 8 \end{bmatrix}$ has $\text{Trace}(\mathsf{M}) =$ ________ and $\text{Det}(\mathsf{M}) =$ ________.

Q9: Wed. 09 Mar $\mu=$ ________ $\leqslant \nu =$ ________ are the eigenvals of $\mathsf{G} := \begin{bmatrix} -5 & -4 \\ 8 & 7 \end{bmatrix}$. Let $\mathsf{D} := \begin{bmatrix} \mu & 0 \\ 0 & \nu \end{bmatrix}$. Then $\mathsf{D} = \mathsf{U}^{-1}\mathsf{G}\mathsf{U}$ where the 2×2 *integer* matrix U is

$$\mathsf{U} = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}.$$

QA: Fri. 11 Mar $\mu=$ ________ $\leqslant \nu =$ ________ are the eigenvals of $\mathsf{G} := \begin{bmatrix} 8 & -12 \\ 9 & -13 \end{bmatrix}$. Let $\mathsf{D} := \begin{bmatrix} \mu & 0 \\ 0 & \nu \end{bmatrix}$. Then $\mathsf{D} = \mathsf{U}^{-1}\mathsf{G}\mathsf{U}$ where the 2×2 *integer* matrix U is

$$\mathsf{U} = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}.$$

QB: Tue. 15 Mar Let $\mathscr{L}$ be this list of 8 symbols: $A, G, R, T, U, \alpha, \beta, \omega$.

In matrix-eqn $\begin{bmatrix} R & U & G \\ 0 & A & 0 \\ 0 & 0 & T \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \alpha \\ \beta \\ \omega \end{bmatrix}$, Cramer's Rule writes x_1 as ratio $f(\mathscr{L}) \big/ q(\mathscr{L})$ of polynomials

$f(\mathscr{L}) =$ ________

and $q(\mathscr{L}) =$ ________.

QC: ?? Fri. 18 Mar Solve the the World's Problems.