

DfyQ
MAP2302

Quizzes Q

08Dec2016

Number Sets. Expression $k \in \mathbb{N}$ [read as “ k is an element of \mathbb{N} ” or “ k in \mathbb{N} ”] means that k is a natural number; a **natnum**. Expression $\mathbb{N} \ni k$ [read as “ \mathbb{N} owns k ”] is a synonym for $k \in \mathbb{N}$.

\mathbb{N} = natural numbers = $\{0, 1, 2, \dots\}$.

\mathbb{Z} = integers = $\{\dots, -2, -1, 0, 1, \dots\}$. For the set $\{1, 2, 3, \dots\}$ of positive integers, the **posints**, use \mathbb{Z}_+ . Use \mathbb{Z}_- for the negative integers, the **negints**.

\mathbb{Q} = rational numbers = $\{\frac{p}{q} \mid p \in \mathbb{Z} \text{ and } q \in \mathbb{Z}_+\}$.

Use \mathbb{Q}_+ for the positive rationals and \mathbb{Q}_- for the negative rationals.

\mathbb{R} = reals. The **posreals** \mathbb{R}_+ and the **negreals** \mathbb{R}_- .

\mathbb{C} = complex numbers, also called the **complexes**.

For $\omega \in \mathbb{C}$, let “ $\omega > 5$ ” mean “ ω is real and $\omega > 5$ ”.

[Use the same convention for $\geq, <, \leq$, and also if 5 is replaced by any real number.]

An “**interval of integers**” $[b..c)$ means the intersection $[b, c) \cap \mathbb{Z}$; ditto for open and closed intervals. So $[e..2\pi] = \{3, 4, 5, 6\} = [3..6] = (2..6]$. We allow b and c to be $\pm \infty$; so $(-\infty..-1]$ is \mathbb{Z}_- . And $[-\infty..-1]$, is $\{-\infty\} \cup \mathbb{Z}_-$.

Floor function: $\lfloor \pi \rfloor = 3, \lfloor -\pi \rfloor = -4$.

Ceiling fnc: $\lceil \pi \rceil = 4$. Absolute value: $|-6| = 6 = |6|$ and $|-5 + 2i| = \sqrt{29}$.

Mathematical objects. Seq: ‘sequence’. poly(s): ‘polynomial(s)’. irred: ‘irreducible’. Coeff: ‘coefficient’ and var(s): ‘variable(s)’ and parm(s): ‘parameter(s)’. Expr.: ‘expression’. Fnc: ‘function’ (so ratfnc: means rational function, a ratio of polynomials). cty: ‘continuity’. cts: ‘continuous’. diff’able: ‘differentiable’. CoV: ‘Change-of-Variable’. Col: ‘Constant of Integration’. Lol: ‘Limit(s) of Integration’. RoC: ‘Radius of Convergence’.

Soln: ‘Solution’. Thm: ‘Theorem’. Prop’n: ‘Proposition’. CEX: ‘Counterexample’. eqn: ‘equation’. RhS: ‘RightHand Side’ of an eqn or inequality. LhS: ‘left-hand side’. Sqrt or Sroot: ‘square-root’, e.g, “the sroot of 16 is 4”. Ptn: ‘partition’, but pt: ‘point’, as in “a fixed-pt of a map”.

FTC: ‘Fund. Thm of Calculus’. IVT: ‘intermediate-Value Thm’. MVT: ‘Mean-Value Thm’.

The **logarithm** fnc, defined for $x > 0$, is $\log(x) := \int_1^x \frac{dv}{v}$. Its inverse-fnc is $\exp()$. For

$x > 0$, then, $\exp(\log(x)) = x = e^{\log(x)}$. For real t , naturally, $\log(\exp(t)) = t = \log(e^t)$.

PolyExp: ‘Polynomial-times-exponential’; e.g, $[3 + t^2] \cdot e^{4t}$. PolyExp-sum: ‘Sum of polyexps’. E.g, $f(t) := 3te^{2t} + [t^2] \cdot e^t$ is a polyexp-sum.

Prefix nt- means ‘non-trivial’. E.g “a nt-soln to $f' = 5f$ is $f(t) := e^{5t}$; a trivial soln is $f \equiv 0$.”

Phrases. WLOG: ‘Without loss of generality’. TFAE: ‘The following are equivalent’. ITOF: ‘In Terms Of’. OTForm: ‘of the form’. FTSOC: ‘For the sake of contradiction’. Use iff: ‘if and only if’.

IST: ‘It Suffices to’ as in ISTShow, ISTExhibit.

Use w.r.t: ‘with respect to’ and s.t: ‘such that’.

Latin: e.g: *exempli gratia*, ‘for example’. i.e: *id est*, ‘that is’. N.B: *Nota bene*, ‘Note well’. QED: *quod erat demonstrandum*, meaning “end of proof”.

Factorial. Def: $n! := n \cdot [n-1] \cdot [n-2] \cdots 2 \cdot 1$; so $0! = 1$.

Rising Fctrl: $\llbracket x \uparrow K \rrbracket := x \cdot [x+1] \cdot [x+2] \cdots [x+[K-1]]$,

Falling Fctrl: $\llbracket x \downarrow K \rrbracket := x \cdot [x-1] \cdot [x-2] \cdots [x-[K-1]]$,

for natnum K and $x \in \mathbb{C}$. E.g, $\llbracket K \downarrow K \rrbracket = K! = \llbracket 1 \uparrow K \rrbracket$.

N.B: For $n \in \mathbb{N}$: If $K > n$ then $\llbracket n \downarrow K \rrbracket = 0$.

Note $\llbracket x \uparrow K \rrbracket = \llbracket x + [K-1] \downarrow K \rrbracket$.

DfyQ [2018t] quizzes so far...

Q1: ^{Fri.}_{31 Aug} A particular polynomial soln $p=p(t)$ to

$$p' + 2p = 6t^2 + 8t + 1$$

is $p(t) = \underline{\hspace{2cm}} \cdot t^2 + \underline{\hspace{2cm}} \cdot t + \underline{\hspace{2cm}}$.

The general soln is $y_\alpha(t) = \alpha e^{Mt} + p(t)$, where $M = \underline{\hspace{2cm}}$. [Put correct numbers in the four blanks.]

Q2: ^{Fri.}_{01 Feb} Blanks $\in \mathbb{R}$. So $\frac{1}{2+3i} = \underline{\hspace{2cm}} + i \cdot \underline{\hspace{2cm}}$.

Thus $\text{Im}\left(\frac{5-i}{2+3i}\right) = \underline{\hspace{2cm}}$.

By the way, $|5-3i| = \underline{\hspace{2cm}}$.

Q3: ^{Wed.}_{06 Feb} For $t > 0$, fnc $y_a(t) := \dots$
 is the gen.soln to $ty' + 2y = t^4$. [Hint: FOLDE. The
 coefficient and target fncs are $C(t) := \frac{2}{t}$ and $G(t) := t^3$.]

Q4: ^{Mon.}_{11 Feb} DE $[\mathcal{N}(x, y) \cdot \frac{dy}{dx}] + \mathcal{M}(x, y) = 0$ is *exact*,
 where
 $\mathcal{N}(x, y) := [x^2 - 7]$ and $\mathcal{M}(x, y) := 2xy + 3e^{3x}$.
 Its solns $y=y(x)$ satisfy $\mathbf{F}(x, y(x)) = \text{Const}$, where
 $\mathbf{F}(x, y) = \dots$

Q5: ^{Wed.}_{13 Feb} DE $[(2x + 8)y \cdot \frac{dy}{dx}] + 4y^2 = 0$ is not, alas, *exact*. Happily, multiplying both sides by (non-constant) fnc $V(y) =$

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 gives a *new* DE which is exact.

Solving the exact-DE, every (non-zero) soln $y=y(x)$ satisfies $F(x, y(x)) = \alpha$, for some constant α , where

$F(x, y) =$

Q6: ^{Mon.}_{25 Feb} Bacteria with birth-multiplier $\mathbf{B} :: \frac{1}{\text{min}}$ are in a petri dish with carrying capacity $\mathbf{C} :: \text{oz}$. The population, $p(t) :: \text{oz}$, satisfies the Logistic DE.

The DE is $\dots\dots\dots$.

For *Hysteria* bacteria, $\mathbf{B} = \frac{1/5}{\text{min}}$. This petri dish has $\mathbf{C}=16\text{oz}$, with initial population $\mathbf{p}_0 = 2\text{oz}$. The time when *Hysteria* has reached half the carrying capacity

is $\dots\dots\dots$ min \approx $\dots\dots\dots$ min.

[NB: You may use $\exp()$ and $\log()$ to express your answer.]

Q7: ^{Fri.}_{01 Mar} Note $[1 + i]^{86} = \left[\dots \right] + i \cdot \left[\dots \right]$.
 [Hint: Multiplying complexes multiplies their moduli, and adds their angles.]

Q8: ^{Wed.}_{17 Apr} The Laplace transform of the function $f(t) := \cos(7t)$ is $\widehat{f}(s) = \dots$.

Two bonus quizzes to be added to Class-T exam.

T1: ^{Fri.}_{15 Mar} With $\mathbf{1}()$ the constant-1 fnc and $F(x) := e^{2x} + \cos(5x)$, then, convolution $[\mathbf{1} \otimes F](x) = \dots$
 $= \frac{1}{2}[e^{2x} - 1] + \frac{1}{5}\sin(5x)$.

Q9: ^{Mon.}_{22 Apr} U.F $y=y(t)$ satisfies $y'' + y' + y = \cos(3t)$, with $y(0) = 5$ and $y'(0) = 7$. Then $\widehat{y}(s) = \dots$.

Differential eqns involve derivatives: T F

T2: ^{Wed.}_{20 Mar} With $G(t) := \cos(\cos(\cos(t)))$, a solution to $2y'' - 7y' + 3y = G$ is $y := f \otimes G$, where $f(t) = \dots$.

GP: ^{Wed.}_{24 Apr} **Games Party!** *at Pascal's Cafe.*