Calc 3
MAC2313 **Quizzes Q** 13Sep2012

Abbrevs. WLOG: '*Without loss of generality*'. TFAE: '*The following are equivalent*'. ITOf: '*In Terms Of*'. OTForm: '*of the form*'. FTSOC: '*For the sake of contradiction*'. Use iff: '*if and only if*'.

IST: '*It Suffices to*' as in ISTShow, ISTExhibit.

Use w.r.t: '*with respect to*' and s.t: '*such that*'.

Latin: e.g: *exempli gratia*, '*for example*'. i.e: *id est*, '*that is*'. N.B: *Nota bene*, '*Note well*'. QED: *quod erat demonstrandum*, meaning "end of proof".

Q1: Wedn. 05 Sep In $\mathbb{R}^3$, the closest point to $\mathbf{v} := (1,2,3)$ on the line through $\mathbf{0}$ and $\mathbf{q} := (-2,7,0)$, is $\alpha\mathbf{q}$, where $\alpha=$ ______

In $\mathbb{R}^2$, with $\mathbf{s} := (1,8)$ and $\mathbf{w} := (4,-2)$, compute $\text{Orth}_{\mathbf{w}}(\mathbf{s})=$ ______.

Q2: Wed. 12 Sep [a] Consider distinct $\mathbf{p},\mathbf{q} \in \mathbb{R}^3$. Using set-builder notation,
$\text{Line}(\mathbf{p},\mathbf{q})=$ ______.

[b] The unit 2-sphere, $\mathbb{S}^2 \stackrel{\text{def}}{=} \{\mathbf{u} \in \mathbb{R}^3 \mid \langle \mathbf{u},\mathbf{u}\rangle = 1^2\}$, is a convex set. *T* *F*

Q3: Fri. 14 Sep In $\mathbb{R}^3$, let $\mathbf{u} := (3,0,-2)$, $\mathbf{v} := (1,1,1)$ and $\mathbf{w} := (3,-3,-7)$. Circle: Then $\mathbf{w} \in \text{Line}(\mathbf{u},\mathbf{v})$: *T* *F*

Then $\mathbf{w} \in \text{Span}(\mathbf{u},\mathbf{v})$: *T* *F*

Then $\text{Span}(\mathbf{u},\mathbf{v},\mathbf{w}) = \mathbb{R}^3$: *T* *F*

Q4: Wed. 19 Sep Cross-product $[2\hat{\imath} + 3\hat{\jmath} - \hat{\mathbf{k}}] \times [3\hat{\imath} + 5\hat{\mathbf{k}}]$ equals ______ $\hat{\imath}$ + ______ $\hat{\jmath}$ + ______ $\hat{\mathbf{k}}$.

For $\mathbf{p},\mathbf{v} \in \mathbb{R}^3$: $\mathbf{p}\times\mathbf{v} = \mathbf{v}\times\mathbf{p}$. *AT* *AF* *Nei*

For $\mathbf{u},\mathbf{q} \in \mathbb{R}^3$: $[\mathbf{u} \times 2\mathbf{q}] \perp 3\mathbf{u}$. *AT* *AF* *Nei*

Q5: Fri. 21Sep Let $\mathbf{p} := (2,1,2)$, $\mathbf{q} := (1,2,3)$ and $\mathbf{r} := (1,1,1)$. Then $\mathbf{n} := [\mathbf{p}-\mathbf{r}]\times[\mathbf{q}-\mathbf{r}]=$ ______. The pt on $\text{Plane}(\mathbf{p},\mathbf{q},\mathbf{r})$ closest to $\hat{\mathbf{0}}$ is ______.

Q6: Tue. 25 Sep Prof. King believes that writing in complete, coherent sentences is crucial in communicating Mathematics, improves posture, and whitens teeth. Circle one:

True! **Yes!** ***What's a sentence?***

A multivariate polynomial, where each monomial has the same degree, is circle

monogamous	**atrocious**		**gregarious**
monic	**expialadocious**		**homogeneous**
manic	**unitary**	**Unitarian**	**utilitarian**

Q7: Fri. 28 Sep Consider sets Ω and B and fnc $f:\Omega\to B$. Then $\text{Graph}(f)=$ ______ (Set-builder notation).

Suppose $g:\mathbb{R}^L\to\mathbb{R}^N$ is continuous. Then $\text{Dim}(\text{Graph}(g)) =$ ______.

For points $\mathbf{T}_k,\mathbf{S} \in \mathbb{R}^3$, the phrase "$[\lim_{k\to\infty} \mathbf{T}_k] = \mathbf{S}$" means $\lim_{k\to\infty}$ ______ = ______.

Q8: Wed. 03 Oct The matrix-product $\begin{bmatrix} 2 & 1 & 4 \\ 0 & -1 & 1 \\ -2 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 \\ 3 & 0 \\ 2 & -1 \end{bmatrix}$ equals ______

Q9: Mon. 26 Nov Inside polar-curve $\text{RadialDist}(\theta) = \theta^3$, the area from ray $\theta = 0$ to $\theta = \frac{\pi}{2}$ is ______.

QA: Fri. 30 Nov The gradient of $h(x,y) := 2^x y + \sin(xy^2)$ is ______ $\hat{\imath}$ + ______ $\hat{\jmath}$.

QB: Tue. 04 Dec *Are you in class today?* ANSWER: "*Of course!*"

§A Potential quiz problems

Some of these may appear on quizzes; naturally, with different data. *Write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.* [N.B: Question numbers starting with 'T*' are too long for microquizzes, but might appear on exams.]

Note: The next few questions may appear on nearby microquizzes, but the later questions need us to study further material first.

Line1: In $\mathbb{R}^3$, let $\mathbf{u} := (8,5,-1)$, $\mathbf{v} := (1,0,-4)$ and $\mathbf{w} := (13,10,10)$. Circle: Then $\mathbf{w} \in \text{Line}(\mathbf{u},\mathbf{v})$: *T* *F*

Then $\mathbf{w} \in \text{Span}(\mathbf{u}, \mathbf{v})$: *T* *F*
Then $\text{Span}(\mathbf{u}, \mathbf{v}, \mathbf{w}) = \mathbb{R}^3$: *T* *F*

CS1: Use $\langle \cdot, \cdot \rangle$ for the inner-product on $\mathbb{R}^N$. State the Cauchy-Schwarz Inequality Thm, fully stating the IFF-condition for equality.

blank lines].

TI1: State the Triangle-inequality thm, fully stating the IFF-condition for equality. [Imagine 5 blank lines].

Ang1: A pyramid over a 100ft×100ft base has height 20ft. Then $\cos(\beta)$= ______, where β is the angle between an apex-edge and a base-edge.

L2: The point $P := (5, \text{-}2)$, in the plane, has orthogonal projection $Q := ($ ______ , ______ $)$ on $\mathbb{L}$, the line $y = 1 + 3x$. [*Check* that $Q \in \mathbb{L}$ and $[P - Q] \perp \mathbb{L}$.]

L3: In $\mathbb{R}^3$, the point $P := (2, \text{-}1, 3)$ has orthogonal projection $\text{Proj}(P)$=(____ , ____ , ____) on the line passing through $(2, 2, 4)$ and the origin.

IP: Points $\mathbf{B}$,$\mathbf{C} \in \mathbb{R}^3$ are distinct. Show that the set $\Omega := \{\mathbf{v} \in \mathbb{R}^3 \mid \langle \mathbf{v} - \mathbf{B}, \mathbf{v} - \mathbf{C} \rangle = 5\}$ is a sphere. Its radius, r, satisfies r^2= ______ (ITOf $\mathbf{B}, \mathbf{C}, 5$).

CS&TI: Triangle Inequality Thm: *For all* $\mathbf{u}$,$\mathbf{w} \in \mathbf{V}$, *we have that* $\|\mathbf{u} + \mathbf{w}\| \leqslant \|\mathbf{u}\| + \|\mathbf{w}\|$. (Omitted: The IFF-condition for equality.)
Derive the Triangle-Inequality Thm, using the C-S thm.

Cvx1: Give an example of a non-convex subset, Ω, of $\mathbb{R}^1$. [Imagine 4 blank lines].

SB1: In $\mathbb{R}^3$, consider the closed radius=9 ball centered at $\widehat{\mathbf{0}}$; let Λ be its complement. Using set-builder notation, write $\Lambda = \{?? \mid ??\}$: [Imagine 2 blank lines].

IP1: State the properties of an inner-product $\langle \cdot, \cdot \rangle$ on $\mathbb{R}^N$. [Imagine 6 blank lines].

Lin1: A fnc $L{:}\mathbb{R}^3 \to \mathbb{R}^2$ is (*strictly*) ***linear*** if (state the two properties): [Imagine 4 blank lines].

LI2: A collection $\mathcal{C} := \{\mathbf{u}_1, \ldots, \mathbf{u}_K\} \subset \mathbb{R}^N$ is ***linearly-independent*** if:

blank lines].