

Number Sets. Expression $k \in \mathbb{N}$ [read as “ k is an element of \mathbb{N} ” or “ k in \mathbb{N} ”] means that k is a natural number; a **natnum**. Expression $\mathbb{N} \ni k$ [read as “ \mathbb{N} owns k ”] is a synonym for $k \in \mathbb{N}$.

\mathbb{N} = natural numbers = $\{0, 1, 2, \dots\}$.

\mathbb{Z} = integers = $\{\dots, -2, -1, 0, 1, \dots\}$. For the set $\{1, 2, 3, \dots\}$ of positive integers, the **posints**, use \mathbb{Z}_+ . Use \mathbb{Z}_- for the negative integers, the **negints**.

\mathbb{Q} = rational numbers = $\{\frac{p}{q} \mid p \in \mathbb{Z} \text{ and } q \in \mathbb{Z}_+\}$. Use \mathbb{Q}_+ for the positive rationals and \mathbb{Q}_- for the negative rationals.

\mathbb{R} = reals. The **posreals** \mathbb{R}_+ and the **negreals** \mathbb{R}_- .

\mathbb{C} = complex numbers, also called the **complexes**.

For $\omega \in \mathbb{C}$, let “ $\omega > 5$ ” mean “ ω is real and $\omega > 5$ ”.

[Use the same convention for $\geq, <, \leq$, and also if 5 is replaced by any real number.]

Use $\bar{\mathbb{R}} = [-\infty, +\infty] := \{-\infty\} \cup \mathbb{R} \cup \{+\infty\}$, the **extended reals**.

An “*interval of integers*” $[b..c]$ means the intersection $[b, c] \cap \mathbb{Z}$; ditto for open and closed intervals. So $[e..2\pi] = \{3, 4, 5, 6\} = [3..6] = (2..6]$. We allow b and c to be $\pm\infty$; so $(-\infty..-1]$ is \mathbb{Z}_- . And $[-\infty..-1]$, is $\{-\infty\} \cup \mathbb{Z}_-$.

Floor function: $\lfloor \pi \rfloor = 3$, $\lfloor -\pi \rfloor = -4$.

Ceiling fnc: $\lceil \pi \rceil = 4$. Absolute value: $|-6| = 6 = |6|$ and $|-5 + 2i| = \sqrt{29}$.

Mathematical objects. Seq: ‘sequence’. poly(s): ‘polynomial(s)’. irred: ‘irreducible’. Coeff: ‘coefficient’ and var(s): ‘variable(s)’ and parm(s): ‘parameter(s)’. Expr.: ‘expression’. Fnc: ‘function’ (so ratfnc: means rational function, a ratio of polynomials). trnfn: ‘transformation’. cty: ‘continuity’. cts: ‘continuous’. diff’able: ‘differentiable’. CoV: ‘Change-of-Variable’. Col: ‘Constant of Integration’. Lol: ‘Limit(s) of Integration’. RoC: ‘Radius of Convergence’.

Soln: ‘Solution’. Thm: ‘Theorem’. Prop’n: ‘Proposition’. CEX: ‘Counterexample’. eqn: ‘equation’. RhS: ‘RightHand side’ of an eqn or inequality. LhS: ‘lefthand side’. Sqrt or Sqroot: ‘square-root’, e.g, “the sqroot of 16 is 4”. Ptn: ‘partition’, but pt: ‘point’ as in “a fixed-pt of a map”.

Binop: ‘Binary operator’. Binrel: ‘Binary relation’.

FTC: ‘Fund. Thm of Calculus’. IVT: ‘intermediate-Value Thm’. MVT: ‘Mean-Value Thm’.

The **logarithm** function, defined for $x > 0$, is $\log(x) := \int_1^x \frac{dv}{v}$. Its inverse-fnc is **exp()**.

For $x > 0$, then, $\exp(\log(x)) = x = e^{\log(x)}$. For real t , naturally, $\log(\exp(t)) = t = \log(e^t)$.

PolyExp: ‘Polynomial-times-exponential’, e.g, $[3 + t^2] \cdot e^{4t}$. PolyExp-sum: ‘Sum of polyexps’. E.g, $f(t) := 3te^{2t} + [t^2] \cdot e^t$ is a polyexp-sum.

Phrases. WLOG: ‘Without loss of generality’. IFF: ‘if and only if’. TFAE: ‘The following are equivalent’. ITOf: ‘In Terms Of’. OTForm: ‘of the form’. FTSOC: ‘For the sake of contradiction’. And ∇ = “Contradiction”.

IST: ‘It Suffices To’, as in ISTShow, ISTExhibit.

Use w.r.t: ‘with respect to’ and s.t: ‘such that’.

Latin: e.g: *exempli gratia*, ‘for example’. i.e: *id est*, ‘that is’. N.B: *Nota bene*, ‘Note well’. *inter alia*: ‘among other things’. QED: *quod erat demonstrandum*, meaning “end of proof”.

Quizzes so far...

S1: Stmt $H \Rightarrow C$ has *contrapositive* $\neg C \Rightarrow \neg H$ and *converse* $\neg H \Rightarrow \neg C$. Recall \wedge, \vee, \neg mean AND, OR, NOT.

Using only symbols $\wedge, \vee, \neg, C, H, [],$, write $H \Rightarrow C$ as $\neg H \Rightarrow \neg C$.

S2: Write the free vars in each of these expressions.

$$\exists n \in \mathbb{N}: f(n) \in \bigcup_{\ell=p-4}^{p+7} \underbrace{\{x \in \mathbb{Z} \mid \ell \cdot n \equiv_5 x^2\}}_{E1} \underbrace{E2}_{E3}$$

E3: $\neg \dots$. E2: $\neg \dots$. E1: $\neg \dots$.

S3: Fri. 28 Feb *What is the day-of-the-week, today?*

$\neg \dots$

S4: Mon. 10 Mar LBolt gives $G := \text{GCD}(23, 413) = \dots$. And $23S + 413T = G$, where $S = \dots$ & $T = \dots$ are integers.