

**Number Sets.** Expression  $k \in \mathbb{N}$  [read as “ $k$  is an element of  $\mathbb{N}$ ” or “ $k$  in  $\mathbb{N}$ ”] means that  $k$  is a natural number; a **natnum**. Expression  $\mathbb{N} \ni k$  [read as “ $\mathbb{N}$  owns  $k$ ”] is a synonym for  $k \in \mathbb{N}$ .

$\mathbb{N}$  = natural numbers =  $\{0, 1, 2, \dots\}$ .

$\mathbb{Z}$  = integers =  $\{\dots, -2, -1, 0, 1, \dots\}$ . For the set  $\{1, 2, 3, \dots\}$  of positive integers, the **posints**, use  $\mathbb{Z}_+$ . Use  $\mathbb{Z}_-$  for the negative integers, the **negints**.

$\mathbb{Q}$  = rational numbers =  $\{\frac{p}{q} \mid p \in \mathbb{Z} \text{ and } q \in \mathbb{Z}_+\}$ . Use  $\mathbb{Q}_+$  for the positive rationals and  $\mathbb{Q}_-$  for the negative rationals.

$\mathbb{R}$  = reals. The **posreals**  $\mathbb{R}_+$  and the **negreals**  $\mathbb{R}_-$ .

$\mathbb{C}$  = complex numbers, also called the **complexes**.

For  $\omega \in \mathbb{C}$ , let “ $\omega > 5$ ” mean “ $\omega$  is real and  $\omega > 5$ ”. [Use the same convention for  $\geq, <, \leq$ , and also if 5 is replaced by any real number.]

Use  $\overline{\mathbb{R}} = [-\infty, +\infty] := \{-\infty\} \cup \mathbb{R} \cup \{+\infty\}$ , the **extended reals**.

An “**interval of integers**”  $[b..c]$  means the intersection  $[b, c] \cap \mathbb{Z}$ ; ditto for open and closed intervals. So  $[e..2\pi] = \{3, 4, 5, 6\} = [3..6] = (2..6]$ . We allow  $b$  and  $c$  to be  $\pm\infty$ ; so  $(-\infty..-1]$  is  $\mathbb{Z}_-$ . And  $[-\infty..-1]$ , is  $\{-\infty\} \cup \mathbb{Z}_-$ .

Floor function:  $\lfloor \pi \rfloor = 3$ ,  $\lfloor -\pi \rfloor = -4$ .  
Ceiling fnc:  $\lceil \pi \rceil = 4$ . Absolute value:  $|-6| = 6 = |6|$   
and  $|-5 + 2i| = \sqrt{29}$ .

**Mathematical objects.** Seq: ‘sequence’. poly(s): ‘polynomial(s)’. irred: ‘irreducible’. Coeff: ‘coefficient’ and var(s): ‘variable(s)’ and parm(s): ‘parameter(s)’. Expr.: ‘expression’. Fnc: ‘function’ (so ratfnc: means rational function, a ratio of polynomials). trnfn: ‘transformation’. cty: ‘continuity’. cts: ‘continuous’. diff’able: ‘differentiable’. CoV: ‘Change-of-Variable’. Col: ‘Constant of Integration’. Lol: ‘Limit(s) of Integration’. RoC: ‘Radius of Convergence’.

Soln: ‘Solution’. Thm: ‘Theorem’. Prop’n: ‘Proposition’. CEX: ‘Counterexample’. eqn: ‘equation’. RhS: ‘RightHand side’ of an eqn or inequality. LhS: ‘lefthand side’. Sqrt or Sqroot: ‘square-root’, e.g., “the sqroot of 16 is 4”. Ptn: ‘partition’, but pt: ‘point’ as in “a fixed-pt of a map”.

Binop: ‘Binary operator’. Binrel: ‘Binary relation’.

FTC: ‘Fund. Thm of Calculus’. IVT: ‘intermediate-Value Thm’. MVT: ‘Mean-Value Thm’.

The **logarithm** function, defined for  $x > 0$ , is  $\log(x) := \int_1^x \frac{dv}{v}$ . Its inverse-fnc is  $\exp()$ .

For  $x > 0$ , then,  $\exp(\log(x)) = x = e^{\log(x)}$ . For real  $t$ , naturally,  $\log(\exp(t)) = t = \log(e^t)$ .

PolyExp: ‘Polynomial-times-exponential’, e.g.,  $[3 + t^2] \cdot e^{4t}$ . PolyExp-sum: ‘Sum of polyexps’. E.g.,  $f(t) := 3te^{2t} + [t^2] \cdot e^t$  is a polyexp-sum.

**Phrases.** WLOG: ‘Without loss of generality’. IFF: ‘if and only if’. TFAE: ‘The following are equivalent’. ITOF: ‘In Terms Of’. OTForm: ‘of the form’. FTSOC: ‘For the sake of contradiction’. And  $\otimes$  = “Contradiction”.

IST: ‘It Suffices To’, as in ISTShow, ISTExhibit.

Use w.r.t: ‘with respect to’ and s.t: ‘such that’.

**Latin:** e.g: *exempli gratia*, ‘for example’. i.e: *id est*, ‘that is’. N.B: *Nota bene*, ‘Note well’. inter alia: ‘among other things’. QED: *quod erat demonstrandum*, meaning “end of proof”.

## Quizzes so far...

**S1:** Stmt  $H \Rightarrow C$  has contrapositive \_\_\_\_\_ and converse \_\_\_\_\_. Recall  $\wedge, \vee, \neg$  mean AND, OR, NOT.

Using *only* symbols  $\wedge, \vee, \neg, C, H, ], [,$ , write  $H \Rightarrow C$  as \_\_\_\_\_.

**S2:** Write the free vars in each of these expressions.

$$\exists n \in \mathbb{N}: f(n) \subset \underbrace{\bigcup_{\ell=p-4}^{p+7} \underbrace{\{x \in \mathbb{Z} \mid \ell \cdot n \equiv x^2\}}_{E1}}_{E2}$$

E3: \_\_\_\_\_ E2: \_\_\_\_\_ E1: \_\_\_\_\_

**S3:** <sup>Fri.</sup><sub>28 Feb</sub> What is the day-of-the-week, to-day?

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**S4:** <sup>Mon.</sup><sub>10 Mar</sub> LBolt gives  $G := \text{GCD}(23, 413) =$  \_\_\_\_\_. And  $23S + 413T = G$ , where  $S =$  \_\_\_\_\_ &  $T =$  \_\_\_\_\_ are integers.