E7, Posting Race. The ***standard ordering*** of the first five primes is $\mathcal{S} := [\![2,3,5,7,11]\!]$. Here is a different ***ordering***: $\mathcal{T} := [\![7,2,11,5,3]\!]$. There are $5! = 120$ orderings.

For $F \in \mathbb{Z}_+$ and an ordering $\mathcal{U}$, let $\mathcal{U}$-name(F) be its tuple of exponents w.r.t $\mathcal{U}$. E.g, $F := 3528360$ factors as $2^3 \cdot 3^6 \cdot 5^1 \cdot 7^0 \cdot 11^2$. So the std-name of F is

$$\mathcal{S}\text{-name}(F) = (3,6,1,0,2). \quad \text{In contrast,}$$
$$\mathcal{T}\text{-name}(F) = (0,3,2,1,6).$$

a Produce an ordering $\mathcal{U}$ and posint N so that

$$\mathcal{U}\text{-name}(\varphi(N)) = \text{UF-zipcode} \stackrel{\text{note}}{=\!=} (3,2,6,1,1),$$

or else prove that no such $\langle \mathcal{U}, N\rangle$ pair exists.

b *Either* exhibit a posint K so that

$$\mathcal{S}\text{-name}(\varphi(K)) = \text{UF-zipcode}$$

or else prove that no such K exists. □

Number Sets. An expression such as $k \in \mathbb{N}$ (read as "k is an element of $\mathbb{N}$" or "k in $\mathbb{N}$") means that k is a natural number; a ***natnum***.

$\mathbb{N}$ = natural numbers = $\{0,1,2,\dots\}$.

$\mathbb{Z}$ = integers = $\{\dots,-2,-1,0,1,\dots\}$. For the set $\{1,2,3,\dots\}$ of positive integers, the ***posints***, use $\mathbb{Z}_+$. Use $\mathbb{Z}_-$ for the negative integers, the ***negints***.

$\mathbb{Q}$ = rational numbers = $\{\frac{p}{q} \mid p \in \mathbb{Z} \text{ and } q \in \mathbb{Z}_+\}$. Use $\mathbb{Q}_+$ for the positive ***ratnums*** and $\mathbb{Q}_-$ for the negative ratnums.

$\mathbb{R}$ = reals. The ***posreals*** $\mathbb{R}_+$ and the ***negreals*** $\mathbb{R}_-$.

$\mathbb{C}$ = complex numbers, also called the ***complexes***.

An "***interval of integers***" $[b \,..\, c)$ means the intersection $[b,c) \cap \mathbb{Z}$; ditto for open and closed intervals. So $[\mathrm{e} \,..\, 2\pi] = \{3,4,5,6\} = [3 \,..\, 6] = (2 \,..\, 6]$. We allow b and c to be $\pm\infty$; so $(-\infty \,..\, -1]$ is $\mathbb{Z}_-$.

Floor function: $\lfloor \pi \rfloor = 3$, $\lfloor -\pi \rfloor = -4$. Ceiling fnc: $\lceil \pi \rceil = 4$. Absolute value: $|-6| = 6 = |6|$ and $|-5+2i| = \sqrt{29}$.

Mathematical objects. Seq: '*sequence*'. poly(s): '*polynomial(s)*'. irred: '*irreducible*'. Coeff: '*coefficient*' and var(s): '*variable(s)*' and parm(s): '*parameter(s)*'. Expr.: '*expression*'. Fnc: '*function*' (so ratfnc: means rational function, a ratio of polynomials). cty: '*continuity*'. cts: '*continuous*'. diff'able: '*differentiable*'. CoV: '*Change-of-Variable*'. CoI: '*Constant of Integration*'. LoI: '*Limit(s) of Integration*'.

Soln: '*Solution*'. Thm: '*Theorem*'. Prop'n: '*Proposition*'. CEX: '*Counterexample*'. eqn: '*equation*'. RhS: '*RightHand Side*' of an eqn or inequality. LhS: '*lefthand side*'. Sqrt **or** Sqroot: '*square-root*', e.g, "the sqroot of 16 is 4". Ptn: '*partition*', *but* pt: '*point*', as in "a fixed-pt of a map".

FTC: '*Fund. Thm of Calculus*'. IVT: '*intermediate-Value Thm*'. MVT: '*Mean-Value Thm*'.

The ***logarithm*** fnc, defined for $x>0$, is $\log(x) := \int_1^x \frac{\mathrm{d}v}{v}$. Its inverse-fnc is $\exp()$. For $x>0$, then, $\exp(\log(x)) = x = \mathrm{e}^{\log(x)}$. For real t, naturally, $\log(\exp(t)) = t = \log(\mathrm{e}^t)$. PolyExp: '*Polynomial-times-exponential*'. E.g, $F(t) := [3+t^2] \cdot \mathrm{e}^{4t}$ is a polyExp.

Phrases. WLOG: '*Without loss of generality*'. TFAE: '*The following are equivalent*'. ITOf: '*In Terms Of*'. OTForm: '*of the form*'. FTSOC: '*For the sake of contradiction*'. Use iff: '*if and only if*'.

IST: '*It Suffices to*' as in ISTShow, ISTExhibit.

Use w.r.t: '*with respect to*' and s.t: '*such that*'.

Latin: e.g: *exempli gratia*, '*for example*'. i.e: *id est*, '*that is*'. N.B: *Nota bene*, '*Note well*'. QED: *quod erat demonstrandum*, meaning "end of proof".

Q1: Tue. 28 Jun May lightning ϟ strike this table! (I.e, please fill in.)

n	r_n	q_n	s_n	t_n
0	98	—	1	0
1	21		0	1
2				
3				
4				

So $\mathrm{Gcd}(98,21) =$ ______ $= [$ ______ $\cdot 98] + [$ ______ $\cdot 21]$.

And $x =$ ______ $\in [0 \,..\, 98)$ solves congr. $21x \equiv_{98} 7$.

Q2: Tue. 05 Jul Mod $K{:=}51$, the recipr. $\left\langle \frac{1}{20} \right\rangle_K =$ ______ $\in [0 \,..\, K)$.

[*Hint:* ↯] So $x=$ ____________ $\in [0..K)$ solves $5 - 20x \equiv_K 2$.

Q3: Wed?. 06 Jul With $N := 19$, then $\varphi(N)=$ ______. Thus EFT (Euler-Fermat) says that $7^{3620} \equiv_N$ ____________ $\in [0..N)$.

Q4: Fri. 08 Jul *Magic integers* G_1, G_2, G_3, each in $[0..330)$, are such that the $g{:}\mathbb{Z}_{11}\times\mathbb{Z}_5\times\mathbb{Z}_6\to\mathbb{Z}_{330}$ mapping is a ring-isomorphism, where

$$g\big((z_1, z_2, z_3)\big) := \Big\langle z_1 G_1 + z_2 G_2 + z_3 G_3 \Big\rangle_{330}.$$

Then $G_3 =$ _________ $\in [0..330)$. Also: "Element $g((5, 6, 11))$ is a zero-divisor in $\mathbb{Z}_{330}$."(Circle) *T* *F*

Q5: Thu. 14 Jul Consider the three congruences C1: $z \equiv_{21} 18$, C2: $z \equiv_{15} 3$, and C3: $z \equiv_{70} 53$. Let z_j be the *smallest natnum* [or *DNE*] satisfying (C1) $\overset{\text{All}}{\ldots}$ (Cj). Then

$z_2=$ ______________________ ; $z_3=$ ______________________ .

Q6: Fri. 15 Jul With $K := 105$, ring $\mathbb{Z}_K$ has $|\mathrm{ZD}_K| =$ ________ and $|\mathrm{NQR}_K| =$ ________.

Q7: Mon. 18 Jul "Integer $49 \in \mathrm{QR}_{91}$" *T F* and "$100 \in \mathrm{QR}_{121}$" *T F*.

Value $K := 857$ is prime. So "$2 \in \mathrm{QR}_K$" *T F* and "$-8 \in \mathrm{QR}_K$". *T F*

The prime decomposition of $L := 22673$ is $7{\cdot}41{\cdot}79$. So "$2 \in \mathrm{QR}_L$". *T F*

Q8: Wed. 20 Jul Suppose $x,y,N \in \mathbb{Z}_+$, with $x^2 + 2y^2 = N$ <u>and</u> $N \perp x$. Thus "*Integer* $-2 \in \mathrm{QR}_N$" is: *T F*

Thus "*Integer* $+2 \in \mathrm{QR}_N$" is: *T F*

With $N \in \mathbb{Z}_+$, consider a $z \in \mathrm{QR}_N$ and a BÃ©zout-pair (U, V) such that $zU + NV = 1$.

Then "*U is a mod-N square*" is: *AT AF Nei*

Q9: Thu. 21 Jul Three Jacobi symbols: Two blanks are immed.:

$\left(\frac{4203}{2006}\right) =$ ________. $\left(\frac{4203}{99}\right) =$ ________. $\left(\frac{120}{7117}\right) =$ ________.

Convolution $[\boldsymbol{\mu} \circledast \boldsymbol{\mu}](12) =$ ________.

[This $\boldsymbol{\mu}$ is the Möbius fnc.]

Oy! *Just as with the Unfinished Symphony, here we have the Unbegin Microquiz. . . .* □

Q *Nein*: Fri. 22 Jul

Integers $\mathbf{13} = 4 + 9$ and $\mathbf{17} = 1 + 16$ are SOTS. So posints $X =$ ____, $Y =$ ____ and $S =$ ____, $T =$ ____, with $X < S < T < Y$, engender these SOTS decompositions: $X^2 + Y^2 = [\mathbf{13} \cdot \mathbf{17}] = S^2 + T^2$.

With $N := 72$: $\varphi(N) =$ ________, $\mathrm{Carm}(N) =$ ________.

QBonus: Tue. 26 Jul State the Quadratic Reciprocity thm, (with all of its hypotheses): ________________________________

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Legendre symbol $\left(\frac{7}{103}\right) =$ ________.

QA: Thu. 28 Jul Polynomial $f(x) := x^2 - x - 22$ has $\mathbb{Z}_2$-root $Y_1 = 0$.

This lifts to $\mathbb{Z}_8$-root $Y_3 =$ ______. And f has a $\mathbb{Z}_5$-root of $Z_1 = -1$, lifting to $\mathbb{Z}_{25}$-root $Z_2 =$ ______.

Magic $G_1 =$ ______, $G_2 =$ ______ realize ring-iso $\mathbb{Z}_8 \times \mathbb{Z}_{25} \to \mathbb{Z}_{200}$, which maps (Y_3, Z_2) to ______, a $\mathbb{Z}_{200}$-root of f.

QB: Tue. 02 Aug Since $221 = 13 \cdot 17$, then $221 = a^2 + b^2$ where $a \perp b$ and posints $a =$ ______ $\leqslant b =$ ______.

Doubling, $442 = x^2 + y^2$, where $x \perp y$ are posints, and $x =$ ______ $\leqslant y =$ ______.

QC: ?? Wed. 03j8 Aug Solve *some* of the World's Problems, involving: *Proof of Fermat SOTS thm, multinomial coeff, finite geom-series, EFT, Wilson's (involution ideas), Stein's problem, Repeated-squaring, CRT, Fusion, Irreducibility/primeness (Chris numbers), compositeness techniques, QuadRecipr* (QR, NQR), *LST* (4NEG, 4POS, 8FAR, *N-RONO*), *JST, Melding (cop-SOTS), Carmichael fnc, Dirichlet convolution* ($\boldsymbol{\mu}, \boldsymbol{\varphi}, Id, \delta, \boldsymbol{\tau}, \boldsymbol{\sigma}$, *Liouville*), *Necklace counting, LCM Lemma, Primroot Thm, Prime-squared Thm, Primroot Lifting Thm, Struc.-of-*$\Phi(2^N)$, *Index-w.r.t-a-generator, Hensel's –and their Progeny, Relatives, and Groupies.*

QD: ?? Thu. 04j8 Aug Solve *two* of the World's Problems.