ACT
MAA4211 7222

Project-P

Prof. JLF King
31Aug2015

Please LaTeX grammatical triple–spaced essays solving the problems. This final-project is due by **4PM, Friday, 24Apr2009**, slid completely under my office door, LIT402.

P1: Disjoint sets $A, B \subset \mathbb{R}$ are each dense in $\mathbb{R}$. Use $\mathbf{0}$ for the constant-zero fnc, and $\mathbf{1}{=}\mathbf{1}_{\mathbb{R}}$ for constant-one. Suppose that fncs $f_n \in \mathbf{C}(\mathbb{R}{\to}\mathbb{R})$ are such that

$$*: \quad f_n\lfloor_A \xrightarrow{n\to\infty} \mathbf{0}\lfloor_A \quad \text{and} \quad f_n\lfloor_B \xrightarrow{n\to\infty} \mathbf{1}\lfloor_B\,,$$

where each convergence is pointwise.

Use BCT to prove $\exists D \subset \mathbb{R}$, dense, such that for each $z \in D$ the sequence $[n \mapsto f_n(z)]$ is *not* convergent; indeed, its limsup $\geqslant \frac{2}{3}$ and its liminf $\leqslant \frac{1}{3}$.

[*H:* Prove $U_1 := \{x \in \mathbb{R} \mid \exists n \in [1..\infty) \text{ s.t } f_n(x) < \frac{1}{3}\}$ is open.]

How can you strengthen this thm? Prove a nice LEMMA which, used twice, establishes the THEOREM.

P2: Let $\mathbf{A}_\delta$ be the annulus $\{z \in \mathbb{C} \mid 0 < |z| < \delta\}$.

i Suppose coeff-seq $\vec{\mathbf{c}} \subset \mathbb{C}$ is not-all-zero (i.e, there is a natnum K with $c_K \neq 0$) and that power-series

$$*1: \quad h(z) := \sum_{n=0}^{\infty} c_n z^n$$

has $R > 0$, where $R := \mathsf{RoC}(\vec{\mathbf{c}})$. Prove there exists $\delta{>}0$ such that

$$*2: \quad \forall z \in \mathbf{A}_\delta : \quad h(z) \neq 0\,.$$

[*Sugg:* First make a LEMMA handling the $c_0{\neq}0$ case. In the gen case, you may want Same-RoC lem from the *Taylor's* pamphlet.]

ii PSes $f(z) := \sum_{n=0}^{\infty} \alpha_n z^n$ and $g(z) := \sum_{n=0}^{\infty} \beta_n z^n$ each have positive RoC. We have a sequence $\vec{\mathbf{y}}$ of *distinct* complex numbers, with $\lim(\vec{\mathbf{y}}) = 0$, such that $\forall k$: $f(y_k) = g(y_k)$. Prove that $\boxed{\forall n\colon \alpha_n = \beta_n}$.

P3: Show no work.

a Differentiable fncs $f_n\colon [0,1]{\to}\mathbb{R}$, where

$f_n(x) :=$ ...., *uniformly*-conv to $\mathbf{0}$. Yet the seq $(f_n')_{n=1}^{\infty}$ of derivatives does *not* even *ptwise*-converge. Indeed $[\limsup_n f_n'(0)] = +\infty$ and $[\liminf_n f_n'(0)] = -\infty$, since

$f_n'(0) =$ ....

b Let f_n denote the n^{th} Fibonacci number, where $\mathsf{f}_0 = 0$ and $\mathsf{f}_1 = 1$. Power-series $\sum_{n=0}^{\infty} \mathsf{f}_n \cdot [x-8]^n$ has RoC= ....

P1:	___ ___ ___	105pts
P2:	___ ___ ___	105pts
P3:	___ ___	55pts
Poorly stapled, or missing ord, name or honor sig :	___	−15pts
Poorly proofread:	___ ___ ___	−One Million pts
Total:	___ ___ ___	265pts

Print name .... Ord:

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: ....