Table of Laplace transforms

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General Lap xforms. For complex numbers α and β, let $\alpha \succ \beta$ mean $\mathrm{Re}(\alpha) > \mathrm{Re}(\beta)$.

With $\boldsymbol{\mu}$ an arbitrary real number and $f, g \in \mathrm{Ord}(\boldsymbol{\mu})$,[♡1] each formula in the table below holds for *all complex* $s \succ \boldsymbol{\mu}$, unless a trailing note says otherwise.

Natural numbers:	N, j, k.
Real numbers:	F. <u>Positive</u> reals: $7, 9$.
Complex Numbers:	R, Q, B_j.

	h or $h(t)$	$\widehat{h}$ or $\widehat{h}(s)$
C:	$f \circledast g$	$\widehat{f} \cdot \widehat{g}$
Mul:	$f(9t)$	$\frac{1}{9} \cdot \widehat{f}(s/9)$
1:	f'	$s \cdot \widehat{f}(s) - f(0)$
1_N:	$f^{(N)}$	$s^N \widehat{f}(s) - \Big[\sum_{j+k=N-1} s^j \cdot f^{(k)}(0)\Big]$
1^{**}_{Ply}:	$[q(\mathbf{D})](f)$	$q \cdot \widehat{f}$
1_{-1}:	$\int_0^t f(x)\,\mathrm{d}x \overset{\text{note}}{=\!=} [\mathbf{1} \circledast f](t)$	$\widehat{f}(s)/s$
2:	$f(t) \cdot t$	$-[\widehat{f}]'$
2_N:	$f(t) \cdot t^N$	$[-1]^N \widehat{f}^{(N)}$
2_{-1}:	$f(t)/t$	$\int_s^\infty \widehat{f}$
3:	$f(t) \cdot \mathrm{e}^{Rt}$	$\widehat{f}(s-R)$
4:	$f(t-7) \cdot \mathbf{H}(t-7)$	$\mathrm{e}^{-7s} \cdot \widehat{f}(s)$
	$f(t) \cdot \mathbf{H}(t-7)$	$\mathrm{e}^{-7s} \cdot \widehat{f(t+7)}(s)$
5:	f has period 7	$\mathcal{L}_7(f)(s) \big/ [1 - \mathrm{e}^{-7s}]$

The "$\mathbf{D}$" in formula (1_{Ply}) is the differentiation operator. The "q" is a general degree-N polynomial

$$q(s) = B_0 + B_1 s + B_2 s^2 + \cdots + B_N s^N.$$

*This*** *needs that derivative* $f^{(k)}(0)$ *is zero, for each* $k \in [0 \,..\, N)$. In that case, (1_{Ply}) is says that function

$$s \mapsto q(s) \cdot \widehat{f}(s)$$

is the xform of $[B_0 f + B_1 f' + B_2 f'' + \cdots + B_N f^{(N)}]$.

Line (5) arranges for periodic input fncs with

$$\mathcal{L}_7(f)(s) := \int_0^7 \mathrm{e}^{-st} f(t)\,\mathrm{d}t.$$

Formula (5) holds $\forall\, s \succ 0$.

Line (3) is valid for all $s \succ R + \boldsymbol{\mu}$.

Formula (Mul) is valid for all $s \succ 9\boldsymbol{\mu}$.

Specific Lap xforms. Many of the below come from the "General" table, with $f := \mathbf{1}$.

	h or $h(t)$	$\widehat{h}$ or $\widehat{h}(s)$
	1	$1/s$
	t^N	$N!/s^{N+1}$
12:	t^R	$\boldsymbol{\Gamma}(R+1)/s^{R+1}$
	$1/\sqrt{t}$	$\sqrt{\boldsymbol{\pi}}/\sqrt{s}$
	$\sqrt{t}$	$\frac{1}{2}\sqrt{\boldsymbol{\pi}}/s^{3/2}$
13:	$\mathrm{e}^{Rt} \cdot t^N$	$N!/[s-R]^{N+1}$
14:	$\mathrm{e}^{Rt} \cdot \sin(Ft)$	$\frac{F}{[s-R]^2 + F^2}$
	$\mathrm{e}^{Rt} \cdot \cos(Ft)$	$\frac{s-R}{[s-R]^2 + F^2}$

Formula (12) requires $R \succ -1$, as does this defn:

$$\boldsymbol{\Gamma}(R+1) := \int_0^\infty t^R \mathrm{e}^{-t}\,\mathrm{d}t.$$

The (12) formulas hold for all $s \succ 0$.

Formulas (13,14) are valid for all $s \succ R$.

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[♡1] See "exponential order" in the `laplace.xform.latex` file.