

Gambling

Let's play BLACKJACK with these cards:

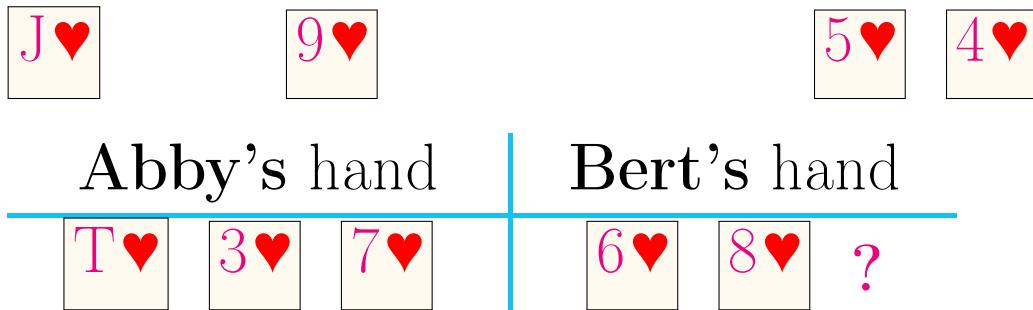


where $J\heartsuit = 11$ and $T\heartsuit = 10$;

Blackjack ($= 21$) is the goal.

Abby and Bert alternate taking a card from the list and putting the card in their hand.

After 5 turns, perhaps the position is this:



A player wins if, after adjoining a card to his hand, he now has some three cards summing to *Blackjack*. If *all* the cards are in player's hands, yet nobody has won, then the game is drawn.

Have You played this game?

Thinking Isomorphically. Suppose we subtract 1 from each card-value. We're now playing with this deck:

T	9	8	7	6	5	4	3	2
♥	♥	♥	♥	♥	♥	♥	♥	♥

Since we need 3 cards to win, our new target is $21 - 3$, i.e 18. This gives us a game which is isomorphic to BLACKJACK.

What happens if we follow this line of thought to its logical conclusion ?

Logical Conclusion. . . . following that line of thought, we iterate the idea. Since $\frac{21}{3} = 7$, subtracting 7 from each card gives symmetry



with 0, now, as the target sum.

But... so what ?

Ringing the Bell. The symmetry suggests something *Magic!* that we've likely seen before...

-3	+4	-1
+2	0	-2
+1	-4	+3

All **eight** TTTs (tic-tac-toes) [three vertical, three horizontal, and two diagonal] sum to 0. And the remaining $\binom{9}{3} - 8 = 84 - 8 = 76$ '**bad**' (non-TTT) triples, do *not* sum to zero. Thus:

*: BLACKJACK is game-isomorphic to TTT.

[Exer: Even though there are 76 bad triples, why do we only need to check 2 of them?]

Soln is temporarily hidden.

Automorphisms. 3×3 -TTT has 8 ***automorphisms*** [self-isomorphisms]. One can check that the only permutations of the 9 cells that preserve TTTs, are the geometric symmetries of a square. This group of 8 is called the ***4th-dihedral group***. It comprises the 4 rotations of the board, together with flipping the board over, then rotating.

[Exer: What is the TTT-automorphism-group of the 4×4 board?
(Shhh..., –are there are some *non-geometric* autos?)]

We can carry these automorphisms back to BLACKJACK, thusly:

$4\heartsuit$	$J\heartsuit$	$6\heartsuit$
$9\heartsuit$	$7\heartsuit$	$5\heartsuit$
$8\heartsuit$	$3\heartsuit$	$T\heartsuit$

BLACKJACK *is suddenly easier to win.*
(Las Vegas, *here I come!*)