Probability II
MAP6473 4810

Homework-1a

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Reading. For Wednesday, 14Jan, please read Billingsley's article "The Singular Function of Bold Play".

Please get your copy of his text as soon as possible.

Please read Sect.7, pages 92–108 by Monday, 19Jan.

Please read part of Sect.8, pages 111–121 by Friday, 23Jan.

1a1: For a random variable Y define

$$\mathcal{N}(\mathsf{Y}) \;:=\; \int_\Omega \mathrm{Min}(|\mathsf{Y}|, 1)\,.$$

Show that

$$\mathcal{N}(\mathsf{Y}) = 0 \iff \mathsf{Y} \stackrel{\text{a.e}}{=} 0\,.$$

Prove or disprove each of the following:

i: $\mathcal{N}(\mathsf{Y}) + \mathcal{N}(\mathsf{Z}) \;\geqslant\; \mathcal{N}(\mathsf{Y}+\mathsf{Z})$.

ii: *If* $\mathcal{N}(\mathsf{Y}_n) \to 0$ then $\vec{\mathsf{Y}}$ converges to 0 in probability.

iii: If $\vec{\mathsf{Y}} \to 0$ in probability, then $\mathcal{N}(\mathsf{Y}_n) \to 0$.

Is *convergence-in-probability* convergence with respect to some metric?

1a2: Consider the following 3-state ***MC*** (Markov chain) with states A, B, C:

State A goes to B and C each with prob$=\frac{1}{2}$.

State B goes to states A, B, C with probabilities $\frac{1}{4}, \frac{1}{4}, \frac{1}{2}$.

State C goes to A, B with probabilities $\frac{1}{3}, \frac{2}{3}$.

Let $\tau(BA)$ denote the transition probability from state B to A.

What is μ, the *unique* invariant distribution? This μ induces a probability on cylinder sets; e.g

$$\mathsf{P}([BBAC]) \;:=\; \mu(B)\cdot\tau(BB)\cdot\tau(BA)\cdot\tau(AC)\,.$$

Please compute this value.

Verify that cylinders $[ABAC]$, $[BBAC]$, $[CBAC]$ have probabilities which sum to $\mathsf{P}([BAC])$.

Verify that cylinders $[BACA]$, $[BACB]$, $[BACC]$ have probabilities which sum to $\mathsf{P}\big([BAC]\big)$.

In detail, verify that the Markov process is *stationary*; you'll need to use that μ is an invariant distribution on the states.