Staple!

Ord: ___ ___

Advanced Calc
MAA4102 & MAA5104

InClass-γ

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γ3: a Please circle those of the following five sets which have the LUB property (each non-void bounded subset has a least upper bound).

Dyadic Rationals, $\mathbb{Q}$, $\mathbb{R}$, $\mathbb{Z} \smallsetminus \{17\}$, $\mathbb{R} \smallsetminus \{17\}$.

___ ___ b *Give an example* of open subsets in $\mathbb{R}$,

$U_n :=$ ...., such that the intersection

$\bigcap_{n=1}^{\infty} U_n =$ .... is *not* open.

___ ___ c In metric space $(\mathbb{Q}, \mathsf{d})$, the set

$$S \;:=\; \{r \in \mathbb{Q} \mid r^2 < 5\}$$

is closed. Circle one: **True** **False**

S is open in $\mathbb{Q}$. **True** **False**

Let f be the indicator function of S, viewed as a function $\mathbb{Q} \to \mathbb{R}$. Then f is continuous. **True** **False**

___ ___ d On your own paper –but show no work– give examples of functions $f, g: \mathbb{R} \to \mathbb{R}$ such that f is continuous **only** at $17 \in \mathbb{R}$, and g is *dis*continuous **only** at $17 \in \mathbb{R}$. [*Hint:* Pictures will help.]

___ ___ e $\lim_{n\to\infty} \left[\frac{2n+1}{2n}\right]^n =$ .... [*Hint:* L'Hôpital's rule.]

___ ___ f Let R_θ be the $2{\times}2$ rotation matrix which rotates the plane counterclockwise by angle θ. So

$$R_\theta \;=\; \begin{bmatrix} & & & \end{bmatrix}.$$

By multiplying matrices $R_\beta R_\alpha$, for angles α and β, compute the sum-of-angle formulae:

$\cos(\beta + \alpha) =$ ....;

$\sin(\beta + \alpha) =$ ....

Note. For the following problems please carefully write up your solutions on separate sheets of paper. ☐

γ4: Let X be the closed interval $[2, 3]$ and suppose that $h: X \to \mathbb{R}$ is continuous. Prove that h is necessarily *uniformly* continuous. You may use, without proof, that X is sequentially compact. [*Hint:* "If h is not uniformly continuous, then there exists a positive number ε such that…".]

γ5: i Give, with proof, an explicit bijection g from $\mathbb{N} \times \mathbb{N}$ onto $\mathbb{N}$.

ii Give, with proof, an explicit bijection h from $3^{\mathbb{N}}$ onto $2^{\mathbb{N}}$.

γ6: Suppose that $f: \mathbb{R} \to \mathbb{R}$ is N-times differentiable, and its N^{th} derivative, $f^{(N)}$, is continuous.

a The N^{th} Taylor polynomial (centered at zero) of f is $\mathbf{T}_N(x) :=$ ....

b Recall that the corresponding remainder term $\mathbf{R}_N(x)$ is $f(x) - \mathbf{T}_N(x)$. The *integral form* of the remainder term is

$\mathbf{R}_N(x) := \int_0^x$ .... $\mathrm{d}t$.

Lagrange's form of the remainder term is "There exists a point c between 0 and x such that $\mathbf{R}_N(x)$ equals ...."

End of InClass-γ

γ3:	___ ___ ___	240pts
γ4:	___ ___ ___	170pts
γ5:	___ ___ ___	170pts
γ6:	___ ___ ___	110pts
Total:	___ ___ ___	690pts

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor (or his colleague)."*
Name/Signature/Ord

Ord: ....