Moda
MAA4227 MAA5229

Project-G

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Carefully TYPE, *double-or-triple–spaced, grammatical essays solving the three problems. This is due by* **3PM, Monday, 25Apr2011**.

Please follow the CHECKLIST *on our* Teaching *page.*

G1: Short answer.

__ __ a) Suppose $f,g\colon\mathbb{R}\to\mathbb{R}$ are each uniformly-cts. Then: *"Composition $f\circ g$ is uniformly-cts."* Circle T F

On $J := (4,5)$ fncs $f,g\colon J\to J$ are unif-cts, and g is bnded. Then *"Pointwise-product $f\cdot g$ is unif-cts."* T F

On $J := (4,5)$ fnc $h\colon J\to J$ is unif-cts and bnded. Then *"h is Lipschitz."* T F

__ __ b) Suppose $h\colon(-2,2)\to\mathbb{R}$ is cts and bnded. Then: *"h is the uniform-limit of polynomials."* Circle T F

A fnc $\varphi\colon[0,1]\to[0,1]$ is ***"good"*** if diff'able, unif-cts and $\varphi(0)=\varphi(1)$. Suppose f_n, g are good, and $\vec{\mathbf{f}}$ *converges-uniformly* to g. Then "$\text{ArcLen}(f_n) \xrightarrow{n\to\infty} \text{ArcLen}(g)$." T F

__ __ c) Suppose $h\colon\mathbb{R}\to\mathbb{R}$ is a non-increasing function. Then: *"The discontinuity set of h is only countable."* T F

__ __ d) Let d be the usual metric on $X := [\mathsf{e},\pi]\smallsetminus\mathbb{Q}$. Then $f(x) :=$ ....

maps $X\to X$, is a (uniform) contraction such that $[\forall y,z\in X\colon \mathsf{d}(f(y),f(z)) \leqslant \frac{1}{2}\cdot\mathsf{d}(y,z)]$, yet f has no fixed-point. This does not contradict our Contraction-mapping thm because

....

G2: With $J := [0,5]$, your goal is to prove:

Thm 1: *Suppose $h\colon J\to\mathbb{R}$ is continuous* [but not-nec differentiable]*. For each $x\in J^\circ$, moreover, $h(x)\neq 0$. If $[h(t)]^2 = [2\cdot\int_0^t h]$ for each $t\in J$, then $h = Id_J$.*

Citing specific thms from class, *rigorously* <u>*prove*</u> that h is non-negative. Now <u>*prove*</u> h differentiable. Then....

G3: On $J := [3,5)$, fnc $f\colon J\to\mathbb{R}$ is cts. Consider this statement: ‡: *$B := \lim_{x\nearrow 5} f(x)$ exists in $\mathbb{R}$*. Prove:

Thm 2a: *If f is <u>uniformly</u>-continuous, then (‡).*

Thm 2b: *If (‡), then f is <u>uniformly</u>-continuous.*

G4: On interval $J := [6,7]$, prove:

Thm 4: *Suppose cts fncs $f,\varphi_n,g\colon J\to\mathbb{R}$ have $\varphi_n \xrightarrow[n\to\infty]{\text{unif}} f$. Then*

$$\int_J g\cdot\varphi_n \xrightarrow[n\to\infty]{} \int_J g\cdot f.$$

Now use the Weierstrass Approx. Thm to carefully prove:

Thm 5: *Fnc $H\colon J\to\mathbb{R}$ is continuous. Moreover, for each $N = 0,1,2,\ldots$, we have that $\left[\int_J H(x)\cdot x^N\,dx\right] = 0$. Then H is the zero-function.*

Explain *carefully/rigorously* <u>how</u> you are applying W.A.T.

The continuity of H is needed at several places in the proof. Explain exactly where, giving rigorous arguments. There is likely a lemma you'll want to state and prove before proving Thm 5.

End of Project-G

	G1:	__ __	95pts
	G2:	__ __	85pts
	G3:	__ __	95pts
	G4:	__ __ __	140pts
Poorly stapled, or missing name, ord or honor signature:		__ __	−15pts
Not double-spaced:		__ __	−15pts
Poorly proofread:		__ __ __	−One Million pts
	Total:	__ __ __	415pts

Please PRINT your name *and* ordinal*. Ta:*

Ord: ....

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: ....

Folks, this full-year course has been quite a pleasure for me, and I hope for you too. We've covered a lot of material, and we have glimpses of a large Mathematical World beyond what we have covered.

Exercise your Mathematical Engines over the summer, and drop-by in the Autumn to let me know what various projects you are working on.

—"Prof. Jonathan"