

**Notes.** Pamphlet *Chromatic polynomial of a graph* on our TEACHING PAGE has definitions. If possible, please use the notation from that pamphlet.

Here, use  $\mathbf{u} - \mathbf{v}$  to mean that vertices  $\mathbf{u}$  and  $\mathbf{v}$  have an edge between them.

**G1:** Show no work. Write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

**a** The vertex set of  $H_N$  is  $\mathbb{V} := [1..3N]$ . For  $\mathbf{u} \in \mathbb{V}$ , when possible:  $\mathbf{u} - [\mathbf{u}+3]$ . If  $\mathbf{u} \equiv_3 1$ , then  $\mathbf{u} - [\mathbf{u}+1]$  and  $\mathbf{u} - [\mathbf{u}+2]$ . If  $\mathbf{u} \equiv_3 2$ , then  $\mathbf{u} - [\mathbf{u}+1]$ . Then

$$\mathcal{P}_{H_N}(x) = \dots$$

**b** For  $N \geq 4$ , let  $D_N$  be  $K_N$  but with an edge (but no vertices) deleted. Then  $\mathcal{P}_{D_N}(x)$  equals

$$\dots$$

**c** The number of spanning-paths in wheel  $W_8$  is

$$\dots$$

OYOP: Your 2 essay(s) must be TYPESET, and Double or Triple spaced. Use the *Print/Revise* cycle to produce good, well thought out, essays. Start each essay on a *new* sheet of paper.

Do not restate the problem; just solve it.

**G2:** Graph  $G$  has  $N$  vertices,  $L$  edges and chrom-poly

$$x^N + B_{N-1}x^{N-1} + B_{N-2}x^{N-2} + \dots + B_1x.$$

Prove that  $B_{N-1} = -L$ .

**G3:** **i** Produce [with proof, naturally] an infinite family of connected graphs which are gluing-good.

**ii** Produce an infinite family of connected graphs which are gluing-bad.

**iii** Can you characterize the family of gluing-good graphs? Conjectures? Numerical evidence? Proofs?

End of Home-G

**G1:** \_\_\_\_\_ 85pts

**G2:** \_\_\_\_\_ 95pts

**G3:** \_\_\_\_\_ 115pts

**Total:** \_\_\_\_\_ 295pts