Staple!

Team: ____

ACT MAA4212 3009 — **Home-G** — Prof. JLF King, *Touch:* 6May2016

Carefully TYPE *triple–spaced, grammatical, learned treatises solving the problems, using* LaTeX. *Due date/time:*

Tuesday, 03Mar2009, by 4PM.

Essays violate the CHECKLIST *at Grade Peril...*

G1: A fnc $f \in \mathsf{Diff}^2(\mathbb{R}\to\mathbb{R})$ with $f(7)=0$ and $f'(7)=0$, satisfies

†: $$f''+f = \mathbf{0}. \quad \text{[I.e, the zero-fnc.]}$$

Prove that $f=\mathbf{0}$, using Taylor's thm, as follows:

First, show that $f \in \mathbf{C}^\infty$. Then, for a *fixed* $x_0 \in \mathbb{R}$, argue that $|f(x_0)|$ is as small as desired, by upper-bounding with Taylor-remainder terms from **Our Taylor's pamphlet** on the Teaching Page.

G2: Dis/Prove: There exists a *series* $\vec{\mathbf{p}} \subset \mathbb{Q}$ which is *absolutely*–$\mathbb{Q}$-convergent, yet $\mathbb{Q}$-divergent. (I.e, sum $\sum_{n=1}^{\infty}|p_n|$ is rational, yet seq $K \mapsto \left[\sum_{n=1}^{K} p_n\right]$ fails to $\mathbb{Q}$-converge.)

Prove an Interesting Lemma that is *more general* than what you need.

G3: Letting $J := [0,1]$, we study fncs $J\to\mathbb{R}$. A ***step function*** is a finite linear-combination of indicator-fncs of intervals.

__ __ **i** For a step fnc $h := \sum_{k=1}^{K} \alpha_k \mathbf{1}_{I_k}$ [where $\alpha_k \in \mathbb{R}$ and each I_k is an interval] and an $\varepsilon>0$, construct a piecewise-linear (P.L fncs are automatically cts) fnc $\varphi() \leqslant h()$ such that $\int_J h \leqslant \varepsilon + \int_J \varphi$. [*Sugg:* As a LEMMA, prove the $K{=}1$ case. Now use the lemma to prove the general THM, arguing *carefully*. For both, draw pictures to illustrate the ideas.]

__ __ **ii** Prove: [Use pics to illustrate your rigorous argument.]

1: Continuous-are-nearby thm. *Fix $f \in \mathbb{R}\mathrm{I}(J\to\mathbb{R})$. Given $\varepsilon>0$, there exists a continuous function $\theta()$, with $\theta \leqslant f$, such that $\int_J f \leqslant \varepsilon + \int_J \theta$.* ◊

__ __ **iii** Use pictures to show how *your construction* works when $f := -\mathcal{R}_{\mathbb{D}}$, the negative of the Ruler-fnc of the dyadic rationals.

G4: Show no work.

a Interval $J := [-3,\boldsymbol{\pi}]$ has ptn P with cutpoints $\{-3,1,\boldsymbol{\pi}\}$. Define $\beta := \left[x \mapsto \sqrt[3]{x}\cdot\mathbf{1}_{\mathbb{Q}}(x)\right]$. Then $\mathsf{Osc}^{\beta}(\mathsf{P})=$ ______ + ______.

Equipping P with sample points $\{-2,\boldsymbol{\pi}/2\}$, now $\mathrm{RS}^{\beta}(\mathsf{P})=$ ______.

b Use $\mathbb{I}\mathrm{RI}$ for "Improper RI". Produce functions $\psi_n \in \mathbb{I}\mathrm{RI}(\mathbb{R}\to\mathbb{R})$ st. $\psi_n \xrightarrow[n\to\infty]{\text{unif}} \mathbf{0}$, yet $\left[\int_{\mathbb{R}}\psi_n\right] \not\to 0$, as $n\to\infty$. Indeed, $\forall n$: $\left[\int_{\mathbb{R}}\psi_n\right] = n^2$. For example, let

$\psi_n :=$ ______.

End of Home-G

G1:	___ ___	75pts
G2:	___ ___	75pts
G3:	___ ___	75pts
G4:	___ ___	40pts
Not in LaTeX:	___ ___ ___	−10pts
Poorly stapled, or missing ordinal :	___	−5pts
Missing name, or honor signature :	___	−5pts
Total:	___ ___ ___	265pts

HONOR CODE: *"I have neither requested nor received help on this exam other than from my team-mates and my professor (or his colleague)."* *Name/Signature/Ord*

______ Ord:

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