Staple!

Team: ____

ACT
MAA4212 3009

Home-F

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Touch: 6May2016

Carefully TYPE *a triple–spaced, grammatical, essay solving the problem. I suggest* LaTeX. *Due date/time:* **The beginning of class, Friday, 16Jan2009**.
Essays violate the CHECKLIST *at Grade Peril...*

F1: Below, X is a MS. The Setting for all parts:

$*$: *Fncs* $g, f_n : X \to \mathbb{R}$ *with* $f_n \searrow g$ *pointwise.*

Let "$\vec{\mathbf{f}}$ is cts" mean that each f_n is continuous. Use $\|\cdot\|$ for the supremum-norm, $\|\cdot\|_{\sup}$, on fncs.

___ ___ z Please carefully prove t.fol theorem.

1: Nested uniform-conv thm (Nested UC). *Suppose* $(*)$, *with* g *cts and* $\vec{\mathbf{f}}$ *cts. Then* $f_n \xrightarrow[n\to\infty]{uniformly} g$ *if* *either:*

i: *Space* X *is compact,* *or*

ii: $\forall \varepsilon > 0$, $\exists$ *an index* $K = K(\varepsilon)$ *such that the set*

$$B_\varepsilon := \left\{ x \in X \;\middle|\; [f_K - g](x) \geqslant \varepsilon \right\}$$

is compact. ◊

[*Generalizing:* Let $B_{n,\varepsilon}$ denote $\left\{ x \in X \;\middle|\; [f_n - g](x) \geqslant \varepsilon \right\}$.]

[*Hint:* Argue carefully (but **briefly**!) that "WLOG, g is the constant-zero function". I.e, having redefined each f_n, you'll need to show that the new $\vec{\mathbf{f}}$ is continuous, and is nested, and pointwise-converges to $\mathbf{0}$. And you'll NTShow that [new $\vec{\mathbf{f}}$ conv-uniformly to $\mathbf{0}$] implies that [orig $\vec{\mathbf{f}}$ conv-uniformly to g].

Having done this, likely you can argue that the sequence $[n \mapsto \|f_n\|]$ is non-incr. The *crux* of the problem is to prove that these numbers decrease *to zero*(!) Don't wave your hands.]

CEXes. In each of t.fol, you are to construct functions g, $\vec{\mathbf{f}}$ and space X fulfilling $(*)$, yet $\|f_n - g\|$ does **not** go to zero. Indeed, $\|f_n - g\| \geqslant 1$ for each n.

Each soln below should have labeled graphs of its f_n and g functions. Place each graph appropriately in its solution-description; don't just shove all the pictures to the end of your write-up.

___ ___ a Here, X is cpt and $\vec{\mathbf{f}}$ is cts, but g might not be.

___ ___ b Here, X is cpt and $g = \mathbf{0}$ but $\vec{\mathbf{f}}$ is not-nec cts.

___ ___ c Here, $g = \mathbf{0}$ and $\vec{\mathbf{f}}$ is cts, but X is not-nec cpt.

F2: For $N = 1, 2, \ldots$, define $b_N := \frac{i}{\sqrt{N}} \in \mathbb{C}$. Create functions $F_N : \mathbb{C} \to \mathbb{C}$ by

$$F_N(z) := \exp\left(\frac{-N}{1 + Nz^2}\right), \quad \text{for } z \notin \{b_N, -b_N\};$$
$$F_N(\pm b_N) := 0.$$

On the ***punctured complex plane*** $\mathbb{C}^\circ := \mathbb{C} \setminus \{0\}$, let $M(z) := \exp(-1/z^2)$. Define $L\!\restriction_{\mathbb{C}^\circ} := M$ and $L(0) := 0$, analogous to what we did in PROJECT E.

d Function $L()$ is continuous at the origin: *T* *F*

e Dis/Prove that $\vec{F}$ pointwise-converges to L.

f Dis/Prove that $\vec{F}$ uniformly-converges to L.

End of Home-F

F1:	___ ___ ___	120pts
F2:	___ ___	45pts
In LaTeX:	___ ___	10pts
Poorly stapled, or missing ordinal :	___	−5pts
Missing name, or honor signature :	___	−5pts
Total:	___ ___ ___	165pts

HONOR CODE: *"I have neither requested nor received help on this exam other than from my team-mates and my professor (or his colleague)."* *Name/Signature/Ord*

Ord: ....

Ord: ....

Ord: ....