Staple!

Team F

Combinatorics MAD4204 4563 | **Home-F** | Prof. JLF King 14Mar2018

Due **BoC, Monday, 19Mar2018**, Please *fill-in* every *blank* on this sheet. Write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

F1: *Show no work.*

a On vertex-set $\mathbb{V}$ with $\lceil \mathbb{V} \rceil = 5$, there are ..... non-isomorphic forests. [Vertices are *not* labeled.]

b Bipartite graph $K_{3,2}$ is Eulerian. *T F*

c The vertex set of H_N is $\mathbb{V} := [1\,..\,2N]$. For $\mathbf{u} \in \mathbb{V}$, *when possible*: $\mathbf{u}$—[$\mathbf{u}$+2]. *If* $\mathbf{u}$ *is odd, then* $\mathbf{u}$—[$\mathbf{u}$+1]. Chromatic-poly of H_N is

$\mathcal{P}_{H_N}(x) =$ .....

In grammatical English ***sentences****,* TYPE *each essay on every* ***third*** *line* (usually)*, so that I can easily write between the lines.* (Do **not** restate the question.) *Start each essay on a new page.*

F2: A graph G is ***rigid*** if its only automorphism is the identity-auto. Below, N denotes the number of vertices in the graph. Prove your results below, providing good large labeled pictures of all graphs.

i For which $N \geqslant 2$ does there exist a simple rigid graph? Same rigid question, but for loopy multi-graphs.

ii For which $N \geqslant 2$ does there exist a rigid *tree*?

iii Below, ***ITree*** means an infinite tree with denumerably many vertices and edges. A 0-ITree has *no* leaves, and an ∞-ITree has infinitely many leaves.

(Dis)Proof: There exists a *rigid* 0-ITree with all vertices of finite-degree. Same question, but now the vertex-set has the stronger *bounded-degree* property.

(Dis)Proof: There exists a *rigid* ∞-ITree with all vertices of finite-degree. Same question, but now the vertex-set must have *bounded-degree*.

F3: There is an island which, from time immemorial, has been divided into N equal-area farming regions, taking up the whole island. It is also divided into N equal-area hunting tracts, taking up the whole island.

There are N married couples on the island; the wives hunt and the husbands farm. We wish to assign tracts to wives and farms to husbands so that each couple can build a house on territory common to both. Indeed, territory with at least area $\delta_N \cdot \text{Area(Island)}$. Determine the largest $\delta = \delta_N$ which that works for *every* division of the island into tracts/regions. [Marriage lemma]

F4: Create an interesting non-trivial graph-theory problem, then solve it. Extra: Create a graph-theory problem that uses an OGF or EGF in a non-trivial way.

End of Home-F

F1:	___ ___	65pts
F2:	___ ___	80pts
F3:	___ ___	80pts
F4:	___ ___	40pts
Total:	___ ___ ___	265pts