Sets and Logic MHF3202 | **Fantabulous-F** | Prof. JLF King, Tuesday, 20Apr2021

F1: Short answer. Show no work.

Write **DNE** if the object does not exist or the operation cannot be performed. NB: **DNE** $\neq \{\} \neq 0$.

a
5 A real number which is *not* transcendental is:

arthritic astringent irrational isometric acerbic chiral pedestrian polyhedral asthmatic algebraic tardy

b
10 10 Each three sets Ω,B,C engender a natural bijection, $\Theta{:}\,\Omega^{B\times C} \hookrightarrow\!\!\!\!\rightarrow [\Omega^B]^C$, defined, for each $f \in \Omega^{B\times C}$, by

$\Theta(f) := \Big[c \mapsto [$ .... $]\Big]$.

Its inverse-map $\Upsilon{:}\,[\Omega^B]^C \hookrightarrow\!\!\!\!\rightarrow \Omega^{B\times C}$ has, for $g \in [\Omega^B]^C$, $\Upsilon(g) := \Big[(b,c) \mapsto [$ .... $]\Big]$.

c
20 Graph G has vertex-set $\mathbb{V} := \{4,5,6,7,8,9\}$. Two vertices $\mathbf{u},\mathbf{v} \in \mathbb{V}$ are connected by an edges IFF difference $|\mathbf{u}-\mathbf{v}|$ is prime. [Neither 0 nor 1 is prime.]

This G is Eulerian *T* *F*

d
10 15 Define $G{:}[1\,..\,12]\circlearrowleft$ where $G(n)$ is the number of letters in the n^{th} Gregorian month. So $G(2) = 8$, since the 2^{nd} month is "February". The only fixed-point of G is ....

The *set* of posints k with $G^{\circ k}(12) = G^{\circ k}(7)$ is ....

[Symbol $G^{\circ k}$ is the ***composition-k^{th}-power*** of G. So $G^{\circ 3}(n)$ means $G\big(G(G(n))\big)$].

[January, February, March, April, May, June, July, August, September, October, November, December]

e
30 Between sets $\mathbf{X} := \mathbb{Z}_+$ and $\mathbf{\Omega} := \mathbb{N}$, consider injections $g{:}\,\mathbf{X} \hookrightarrow \mathbf{\Omega}$ and $h{:}\,\mathbf{\Omega} \hookrightarrow \mathbf{X}$, defined by

$$g(x) := 3x \quad \text{and} \quad h(y) := y + 5\,.$$

Schröder-Bernstein produces a set $B \subset h(\mathbf{\Omega}) \subset \mathbf{X}$ st., letting $F := \mathbf{X} \setminus B$, the fnc $\theta{:}\,\mathbf{X} \hookrightarrow\!\!\!\!\rightarrow \mathbf{\Omega}$ is a *bijection*, where

$$*: \qquad \theta\lfloor_F := g\lfloor_F \quad \text{and} \quad \theta\lfloor_B := h^{-1}\lfloor_B\,.$$

For this (g,h), the (F,B) pair is unique. Computing, $\theta(\mathbf{56})=$ .... $\theta(\mathbf{137})=$ .... $\theta^{-1}(\mathbf{603})=$ ....

Essay. Hand-write your essay on paper, writing large and dark. [You may typeset it if you wish, but that is likely slower.] *Do Not Scrunch!* You can write on every 2^{nd}-line to make your essay easier to read. The essay is written in complete sentences, correctly spelled and punctuated, and assembled into logical paragraphs.

When the essay is done, convert it to a pdf, then name it as follows: `F2.`<your name>`.pdf`

E.g, Rachel Stein will name her pdf file as
`F2.Rachel-Stein.pdf`

There are no spaces in the filename! The extension is "`pdf`". A hyphen is used to separate the given-name from the family-name. (You may write names in the order you are accustomed.) ☐

20 20 15 **F2:** Prove that the map $f{:}\,\mathbb{N}{\times}\mathbb{N} \to \mathbb{Z}_+$, with $f(k,n) := 2^k \cdot [1+2n]$, is *surjective*. Now prove f *injective*. [Start your *Surjectivity argument* with Proof and end with QED. The (separate) *Injectivity argument* starts/ends with PROOF/QED. State *specifically* what part of what theorem you are using, for each proof.]

Value $f^{-1}(928)$ equals ...

5 **F3:** *"I have neither requested nor received help on this exam other than from my professor."*

F1:	___ ___ ___	100pts
F2:	___ ___	55pts
F3:	___	5pts
Total:	___ ___ ___	160pts