Moda
MAA4227 MAA5229

Class-F

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F1: *Show no work. Please write* **DNE** *in a blank if the described object does not exist or if the indicated operation cannot be performed.*

a Let $\mathbf{J} := [0,1]$. A map $h{:}\mathbf{J}\to\mathbb{R}$ is ***Lipschitz cts*** IFF

An example of a *continuous* but **not** Lipschitz $f{:}\mathbf{J}\to\mathbb{R}$, which is *differentiable* on $\mathbf{J}^\circ = (0,1)$ is

$f(x) :=$.

b Suppose function $G{:}[7,9]\to\mathbb{R}$ is discontinuous on a dense set, and is non-increasing. For the statement "*G is Riemann integrable*", circle the best line:

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No-such-functions-exist
Always-True
Sometime-True--Sometimes-False
Always-False
```

Essay questions, triple-spaced. Start each essay on a new sheet of paper.

F2: α For two real normed-vectorspaces $(\mathbf{V}, \|\cdot\|)$ and $(\mathbf{H}, \lceil\cdot\rceil)$, define what it means for a map $L{:}\mathbf{V}\to\mathbf{H}$ to be ***linear***.

β At point $P \in \mathbf{V}$, define precisely what it means for a map $f{:}\mathbf{V}\to\mathbf{H}$ to be **"*differentiable* at P"**. (This is sometimes called the "total derivative".) If your defn needs them, use $\mathbf{0_V}$ and $\mathbf{0_H}$ for the zero-vectors in the two spaces. Use the correct symbol, $\|\cdot\|$ or $\lceil\cdot\rceil$, if you use the norms in your defn.

F3: Fnc $f(x) := x^3 + 3x^2 + x$ mapping $\mathbb{R}\to\mathbb{R}$ has many fixed-points. Classify each as **R** repelling, **A** attracting, **LA-RR** left-attract right-repel, **RA-LR** right-attract left-repel. f–fixed-pts:

$x=$ is: **R A LA-RR RA-LR**

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For each fixed-point, use a full sheet of paper to graph f and the appropriate line on the same coord-system, and draw the Verhulst diagram (cobweb plot) of an f-orbit starting near the fixed-point. Draw arrows, so I see which way is "time-going-forward" on the Verhulst diagram.

F4: i Define a (***uniform***) ***contraction map*** [on what kind of space?] and a ***weak-contraction map***.

ii Carefully state the Contraction mapping theorem, both parts.

iii Prove the Contraction mapping theorem.

End of Class-F

F1:	___ ___	50pts
F2:	___ ___	35pts
F3:	___ ___	65pts
F4:	___ ___	90pts
Poorly stapled, or missing name or honor sig:	___ ___	−15pts
Not triple-spaced:	___ ___	−15pts
Total:	___ ___ ___	240pts

Please PRINT your name and ordinal. Ta:

Ord:

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: