ACES MAA4103 — **Prac-F** — Prof. JLF King 20Feb2009

Hello. *Please write* **DNE** *in a blank if the described object does not exist or if the indicated operation cannot be performed.*

Below, $J := [3,7]$.

F1: Show no work.

z) A multivariate polynomial, where each monomial has the same degree, is [circle]

monogamous **atrocious** **gregarious**
homogeneous **expialadocious** **monic**
manic **unitary** **Unitarian** **utilitarian**

a) P.L fncs f_n converge ptwise, but **not** uniformly, to $x \mapsto 2x$ where the cutpoint and height tuples of f_n are $\vec{\mathbf{p}} := (1, \quad \frac{n+1}{n}, \quad \frac{n+3}{n}, \quad 4)$ and $\vec{\mathbf{h}} := (2, \_\_\_\_, \_\_\_\_, 8 + \frac{1}{n})$.

P.L fncs g_n converge ptwise, but **not** uniformly, to $-Id$ where the cutpoint and height tuples of g_n are

$$\vec{\mathbf{p}} := (2,\ 3, \_\_\_\_, 5)$$

and $$\vec{\mathbf{h}} := (-2, -3, \_\_\_\_, -5).$$

b) A map $f\colon \mathbf{V}\times\mathbf{E}\to\mathbf{W}$ (where $\mathbf{V}, \mathbf{E}, \mathbf{W}$ are $\mathbb{R}$-vectorspaces) is ***bilinear*** if $\forall \mathbf{v}_1, \mathbf{v}_2 \in \mathbf{V}$, $\forall \mathbf{e}_1, \mathbf{e}_2 \in \mathbf{E}$ and

$\forall$ ____ : ____

and ____ .

A map $\langle\cdot,\cdot\rangle$ from $\mathbf{V}\times\mathbf{V}\to\mathbb{R}$ is an ***inner product*** if

I1: ____ .

I2: ____ .

I3: ____ .

c) Writing poly $p(x) := 9 + 27x^2 - 7x^3 + 5x^4$ as $\sum_{k=0}^{4} C_k \cdot [x+2]^k$, coeff C_3 is in: [Circle *one* interval]

$(-\infty, -70)$, $[-70, -15)$, $[-15, -8)$, $[-8, -1)$, $[-1, 8)$, $[8, 15)$, $[15, 30)$, $[30, 75)$, $[75, 94)$, $[94, +\infty)$.

d) Let $h := [y \mapsto \sin(3y)]$. Then the 5-topped poly

$\mathbf{T}^h_{5,0}(x) =$ ____ .

e) Let $\varphi(x,y) := x^3y^5 + 2^x$. Then the Hessian matrix of φ is $H(x,y) =$ ____ (More room than given here.)

f1) On the circle $x^2+y^2 = 1^2$, the max-point of $\Gamma(x,y) := \sin(xy)$ is (____ , ____).

f2) On the ellipse $[\frac{x}{3}]^2 + [\frac{y}{4}]^2 = 1^2$, the max-point of $\Gamma(x,y) := x - 2y$ is (____ , ____).

g) On $J := [3,7]$, we have partition P with cutpoints $p_0 = 3 < p_1 < \cdots < p_9 = 7$. For fnc $g\colon J\to\mathbb{R}$, the defn of $\mathsf{Osc}^g(\mathsf{P})$ is: ____

h) Interval $J := [3,7]$ has ptn Q with cutpoints $\{3,5,7\}$. Define $h := [x \mapsto x \cdot \mathbf{1}_{[5,7)}(x)]$. Then

$\mathsf{Osc}^h(Q) =$ ____ + ____ .

Equipping Q with sample points $\{4,6\}$, now

$\mathsf{RS}^h(Q) =$ ____ .

Essay question: Carefully write a triple–spaced essay solving the problem.

F2: Consider a $\mathbf{C}^\infty$-fnc $h\colon\mathbb{R}\to\mathbb{R}$ and point $Q \in \mathbb{R}$. Formally define (using our notation) $\mathbf{T}^h_{N,Q}(x)$ and the remainder term $\mathbf{R}^h_{N,Q}(x)$.

Now give the MVT-version of a type of formula for $\mathbf{R}^h_{N,Q}(x)$. [*Hint:* Get the quantifiers correct, and specific.]

Now give an *integral-formula* for $\mathbf{R}^h_{N,Q}(x)$.

F3: For a function $f\colon J\to\mathbb{R}$, write the formal ε, δ definition of: "$\left[\int_J f\right] = -4$".

[Extra:]
F4: Give a specific example of a *bounded* fnc $h\colon J\to\mathbb{R}$ which is *not* Riemann integrable, with proof.

End of Prac-F