Staple!

Ord: ___ ___

Combinatorics MAD4204 4563 | **Class-F** | Prof. JLF King Wedn., 21Mar2018

Welcome. Write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

F5: Short answer. Show no work.

a ___ Prof. King thinks that submitting a ROBERT LONG PRIZE ESSAY [typically 2 prizes, $500 total] is a *really good idea*, and the due date for the emailed-PDF is **March 28**, but he'd like a draft a few days earlier. Circle:

Yes **True** **Yeah, man!**

b ___ For $K \geqslant 1$, the K-dim'al ***hypercube-graph*** U_K has 2^K vertices; the bit-sequences of length-K. Two vertices are connected by an edge IFF their bit-seqs differ in a single position. [So U_2 and U_3 are the edge-graphs of the square and cube.] Thus $|\mathrm{Aut}(U_K)| =$ _____.

c ___ The n vertices of a *connected, simple* graph G all have the same degree. When G has 22 edges, the values of n are (list, separated by commas): _____.

OYOP: *In grammatical English* ***sentences****, write your essays on every* ***third*** *line* (usually)*, so that I can easily write between the lines.* Start each essay on a new sheet-of-paper.

F6: For $N \geqslant 3$, let D_N be the complete graph K_N but with one edge removed; so D_N has $\binom{N}{2}-1$ edges. Let $f(N)$ be the number of spantrees of D_N.

i Compute $f(3) =$ _____ by drawing each spantree isomorphism-type and counting how often it occurs. Do the same for $f(4) =$ _____

ii Numbering the vertices $1, 2, \ldots, N$, with the removed-edge being between vertices $N-1$ and N, exhibit (draw it LARGE) the Laplacian matrix $\mathsf{L} = \mathsf{L}(D_N)$.

iii (Draw matrices L_0, H and U, LARGE and unambiguous.) Remove the last row and column from L to produce a reduced-Laplacian L_0.

Replacing the first row by the sum of all the rows, produces matrix H. Adding the first H-row to all the others, creates upper-triangular matrix U. Thus

$\#\mathrm{Spantrees}(D_N) =$ _____.

F7: For $N \geqslant 3$ prove, as we did in class, that an N-vertex di-connected tournament admits a [directed] Hamiltonian cycle.

End of Class-F

F5:	___ ___	50pts
F6:	___ ___	50pts
F7:	___ ___	40pts
Total:	___ ___ ___	140pts

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor (or his colleague)."*
Name/Signature/Ord

Ord: _____ ___ ___