Optional essay question, done individually:

This can raise or lower your course-grade by a half-grade. If you current grade is B+, then a good project earns an A, an acceptable but mediocre project leaves you at B+, and a poor project earns a B.

Fill-in all blanks, and carefully TYPE *a triple–spaced, grammatical, essay solving the problem. I suggest* LaTeX; *you should learn it now, since it will be* **required** *next semester.   This final-project* (if you choose to hand it in) *is due by* **noon, Friday, 12Dec2008**, *slid completely* <u>under</u> *my office door,* LIT402.

Essays violate the <u>CHECKLIST</u> at *Grade Peril…*

E1: For $N = 1, 2, \ldots$, define fncs $R_N, F_N : \mathbb{R} \to \mathbb{R}$ by

$$R_N(x) := \exp\left(\frac{N}{1+Nx^2}\right) \quad \text{and} \quad F_N := 1/R_N.$$

a Prove that, pointwise, $F_1 \geqslant F_2 \geqslant F_3 \geqslant \ldots \geqslant 0$. Use this to prove, $\forall x \in \mathbb{R}$, that $L(x) := \lim_{n \to \infty} F_n(x)$ exists in $\mathbb{R}$. Computing, $L(0) = $ _____ .

b There is a "Calc 1" fnc $M(x) :=$ _____ ,

defined[♡1] on $U := \mathbb{R} \setminus \{0\}$, so that $L|_U = M|_U$. Differentiating, $M'(x) = M(x) \cdot$ _____ .

Graph L and some of the F_n. Indicate horizontal asymptotes, points of inflexion, local max/min-points, TW9Ys.[♡2]

Let $\|\cdot\|$ denote the supremum-norm on functions. Compute $\|L - F_n\|$. *Prove* or *dis*prove that

$$F_n \xrightarrow[n \to \infty]{\text{uniformly}} L. \quad (?)$$

Is the $(F_n)_{n=1}^{\infty}$ sequence $\|\cdot\|$-Cauchy?

c In any case, prove that $L()$ is continuous at zero. Please prove the.fol proposition, which may help with the above and below tasks. (Is L'H useful?)

1: Prop'n.   *For each polynomial $P()$, necessarily* $\lim_{x \to 0} \left[P(\frac{1}{x}) \cdot M(x) \right]$ *equals zero.*   ◇

With this tool in hand, prove that L is differentiable at zero by directly applying the **definition** (difference-quotient) of derivative.

d Now prove that L is ∞ly differentiable by establishing the following nifty lemma.

[♡1]Note that $L()$ and $M()$ are *different* functions; after all, they have different domains.
[♡2]The Whole Nine Yards.

2: Nifty Poly Lemma. *Given a polynomial P, define*

3: $L_P\big(x\big) \quad := \quad \begin{cases} 0, & \text{if } x = 0 \\ P(\frac{1}{x}) \cdot M(x), & \text{if } x \neq 0 \end{cases}.$

Prove that L_P is differentiable at the origin (use the **defn** of derivative), *as well as everywhere else. Give a formula* (in terms of P) *for a new polynomial $\widehat{P}$ satisfying that* $\boxed{\big[L_P\big]' = L_{\widehat{P}}}$. (The foregoing **Proposition** may help in proving equality at zero.)   ◇

At this juncture, let Q_0 be the constant-1 polynomial. Inductively define $Q_{k+1} := \widehat{Q_k}$, for each $k \in \mathbb{N}$. Prove that L is ∞ly differentiable by showing that $L^{(k)} = L_{Q_k}$, for each $k = 0, 1, 2, \ldots$.

Exhibit a table showing the coefficients of polynomial Q_k, for $k = 1, 2, 3, 4$, at least.

(Do you see a pattern, or a recurring property, here?)

End of ACES-E

E1:	__ __	55pts	
Poorly stapled, or missing ordinal :	__	−5pts	
Missing name, or honor sigs :	__	−5pts	
Poorly proofread:	__ __ __	−One Million pts	
Total:	__ __ __	±55pts	

Print name ....   Ord: ....