*Staple!*

| Combinatorics | | Prof. JLF King |
|---|---|---|
| MAD4204 4563 | **Home-E** | 30Jan2018 |

Due **BoC, Monday, 05Feb2018**, Please *fill-in* every *blank* on this sheet. Write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

A perm of cycle-signature $\lceil 7^3, 4^2, 1^8 \rfloor$ has three 7-cycles, two 4-cycles, and eight fixed-pts. Use **CN** for "cycle notation", and **CCN** for "canonical CN".
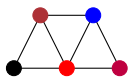
**E1:** *Henceforth, show no work. Simply fill-in each blank on the problem-sheet.*

**a** The physics lab has atomic *zinc, tin, silver* and *gold*. I'm allowed to take 6 atoms, so I have [expressed as single integer] _____ many possibilities.

**b** Perm $\pi := [6,7,8,1,2,3,4,5]$ has $\mathrm{Sgn}(\pi) = $ **+1  −1** .

**c** Perm $\gamma \in \mathbb{S}_{10}$ is $(4\,3\,2\,1\,0)(9\,8\,7\,6\,5)$. The possible cycle-types of $\beta$, where $\beta^2 = \gamma$, are _____ . And our $\gamma$ has _____ many sqroots.

**d** *Trapezoid:* Graph _____ has *Chromatic* poly $\mathcal{P}(x) = $ _____ .

[Edge-glue three copies of $K_3$. Do not multiply out, but do write as a product of polynomials..]

*In grammatical English* sentences, $\mathrm{TYPE}$ *the essay on every* third *line* (usually)*, so that I can easily write between the lines. Do* **not** *restate the question.*

**E2:** With $b_0 := 1$, $b_1 := 0$, $b_2 := 2$, suppose $\forall n \geqslant 3$: $b_n := 2b_{n-1} + 5b_{n-2} - 6b_{n-3}$. Showing all the steps, compute the OGF of $\vec{\mathbf{b}}$. Produce a formula for $b_n$.

**E3:** With $a_0 = a_1 := 1$, imagine that $\forall n \geqslant 2$: $a_n := na_{n-1} + n[n-1]a_{n-2}$. Compute the **EGF** of $\vec{\mathbf{a}}$, with all the details.

**E4:** Create a "story problem" which produces an interesting, non-trivial sequence that can be studied via either an OGF or an EGF, and where our Product-thm or Composition-thm [for OGF or EGF] can be use profitably.

Construct the GF, and use it to compute the $n^{\text{th}}$ term in the sequence. After the fact, can you give a direct combinatorial argument?

| End of Home-E |
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| **E1:** | __ __ | 75pts |
|---|---|---|
| **E2:** | __ __ | 65pts |
| **E3:** | __ __ | 65pts |
| **E4:** | __ __ | 20pts |
| **Total:** | __ __ __ | 225pts |

*Note: Staple this problem-sheet to your essay. The problem-sheet is the first page of what you hand in.*