Staple!

**Ord:** ___ ___

Linear Algebra
MAS4105 *6137*

**Class-E**

Prof. JLF King
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***Hello.*** Every IP is conj-linear in its first argument: So $\langle\alpha\mathbf{u},\mathbf{w}\rangle = \overline{\alpha}\langle\mathbf{u},\mathbf{w}\rangle$ and $\langle\mathbf{u},\alpha\mathbf{w}\rangle = \alpha\langle\mathbf{u},\mathbf{w}\rangle$.

**E1:** Show no work. Please write **DNE** in a blank <u>if</u> the described object does not exist or if the indicated operation cannot be performed.

___ ___ a) A multivariate polynomial, where each monomial has the same degree, is circle

**monogamous** **atrocious** **gregarious**
**monic** **expialadocious** **homogeneous**
**manic** **unitary** **Unitarian** **utilitarian**

___ ___ b) Let $\mathbf{p} := (4,1,3)$, $\mathbf{q} := (1,7,0)$ and $\mathbf{w} := (1,1,0)$. A dir-vec for $\mathbb{L} := \text{Line}(\mathbf{p},\mathbf{q})$ is $\mathbf{D} := \mathbf{p} - \mathbf{q} =$ ______________.

The point on $\mathbb{L}$ closest to $\mathbf{w}$ is $\mathbf{C} =$ ______________.

___ ___ c) Line $y = \big[[\quad\quad]x\big] +$ ______ is the least-squares best-fit to data pts $\big\{(-3,0),(-1,0),(1,0),(3,1)\big\}$.

___ ___ d) Let $\mathsf{B} := \begin{bmatrix} 3 & 6 & 0 & -3 & 0 \\ 1 & 2 & 1 & 0 & 1 \end{bmatrix}$. Then $\mathsf{R} := \mathit{RREF}(\mathsf{B})$ is [show no work, here]

$$\mathsf{R} = \left[ \quad \Big| \quad \Big| \quad \Big| \quad \Big| \quad \right].$$

I) For subspace $\mathbf{V} := \text{Nul}(\mathsf{L}_\mathsf{B})$, use back-substitution, and *scaling*, to produce an *<u>integerized-basis</u>* (i.e, in each vector the Gcd of the entries is 1, and the first non-zero value is positive)

$\mathbf{v}_1 := (\ \_,\ \_,\ \_,\ \_,\ \_\ )$ $\mathbf{v}_2 := (\ \_,\ \_,\ \_,\ \_,\ \_\ )$,
$\mathbf{v}_3 := (\ \_,\ \_,\ \_,\ \_,\ \_\ )$, $\mathbf{v}_4 := (\ \_,\ \_,\ \_,\ \_,\ \_\ )$.

[*Note:* **Only use as many as the dimension of V.**]

___ ___ II) Apply Gram-Schmidt to compute an **orthogonal** *integerized-basis* for $\mathbf{V}$:

$\mathbf{b}_1 := (\ \_,\ \_,\ \_,\ \_,\ \_\ )$, $\mathbf{b}_2 := (\ \_,\ \_,\ \_,\ \_,\ \_\ )$,
$\mathbf{b}_3 := (\ \_,\ \_,\ \_,\ \_,\ \_\ )$, $\mathbf{b}_4 := (\ \_,\ \_,\ \_,\ \_,\ \_\ )$.

___ ___ e) Let $\mathsf{P}: \mathbb{R}^2 \to \mathbb{R}^2$ be orthogonal projection on the $\theta$-angle line. [Picture on blackboard.] In terms of numbers $\mathbf{c} := \cos(\theta)$ and $\mathbf{s} := \sin(\theta)$, then,

w.r.t the std-basis, $[\![\mathsf{P}]\!]_{\mathcal{E}}^{\mathcal{E}}$ equals $\begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}$.

f) Linear $\mathsf{S},\mathsf{T}: \mathbf{V} \to \mathbf{V}$ on $\mathbb{C}$-IPS with $\text{Dim}(\mathbf{V}) = 20$. Use $\text{Geo}(\alpha,\mathsf{T})$ and $\text{Alg}(\alpha,\mathsf{T})$ for the geometric and algebraic multiplicities of $\alpha \in \mathbb{C}$, for $\mathsf{T}$.

1) If $2, -2, 2\boldsymbol{i}$ are $\mathsf{T}$-$\mathcal{e}$vals, then $\text{Geo}(16, \mathsf{T}^4) \geqslant 3$. *T* *F*

2) If $\beta$ is a $\mathsf{T}$-$\mathcal{e}$val and $h$ is an arbitrary $\mathbb{C}$-polynomial, then $h(\beta)$ is an $\mathcal{e}$val for $h(\mathsf{T})$. *AT AF Nei*

3) If $\alpha$ is an $\mathcal{e}$val for both $\mathsf{T}$ and $\mathsf{S}$, then $\alpha$ is an $\mathsf{ST}$-$\mathcal{e}$val. *AT AF Nei*

4) If $7\boldsymbol{i} - 5$ is an $\mathcal{e}$val for $\mathsf{T}^*$, then $7\boldsymbol{i} + 5$ is an $\mathcal{e}$val for $\mathsf{T}$. *AT AF Nei*

OYOP: Essay: *Write on every* ***third*** *line, so that I can easily write between the lines.*

**E2:** On $\mathbb{C}$-IPS, consider a linear fnc'al $h: \mathbf{V} \to \mathbb{C}$. Suppose that $\text{Dim}(\mathbf{V}) = 6$. Prove $\exists\mathbf{u} \in \mathbf{V}$, such that $h(\cdot) = \langle\mathbf{u}, \cdot\rangle$.

End of Class-E

| | | |
|---|---|---|
| **E1:** | ___ ___ ___ | 170pts |
| **E2:** | ___ ___ | 45pts |
| **Total:** | ___ ___ ___ | 215pts |