

Welcome. Write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed. **Write expressions unambiguously** e.g, “ $1/a + b$ ” should be bracketed either $[1/a] + b$ or $1/[a + b]$. (Be careful with negative signs!)

Recall $\llbracket x \downarrow K \rrbracket := x \cdot [x-1] \cdot [x-2] \cdots [x-[K-1]]$ is read as “ N falling-factorial K ”.

E5: Short answer. Show no work.

a Prof. King believes that writing in complete, coherent sentences is crucial in communicating Mathematics, improves posture, and whitens teeth. Circle one:

True! Yes! wH'at S a? sEnTENcE

b The physics lab has atomic *zinc*, *tin*, *silver* and *gold*. I'm allowed to take 7 atoms, so I have [expressed as single integer] many possibilities.

.....

c ITOf binomial coeffs, the n^{th} Catalan-# is $c_n =$.
And $c_5 =$.

d For posint N , let G_N be K_N (complete graph) but with one edge removed (so G_N has N vertices). Its chromatic polynomial

is $\mathcal{P}_{G_N}(x) =$.

[You may use rising/falling factorial in your answer, if you wish.]

In particular, $\mathcal{P}_{G_4}(x) =$.

e Writing $1/[1 + 2x]^8$ as $\sum_{n=0}^{\infty} a_n \cdot x^n$, then $a_5 = M \cdot \binom{U}{D}$
where $M =$, $U =$, $D =$.

f Suppose $G(x)$ is the OGF of seq. $\vec{p} = (p_0, p_1, \dots)$, where p_n is the number of partitions of n whose parts are primes < 9 .
Then $G(x) =$.

[You may leave $G(x)$ as a product of rational fncs.]

OYOP: In grammatical English **sentences**, write your essay on every **third** line (usually), so that I can easily write between the lines. Please number the pages “1 of 57”, “2 of 57”... (or “1/57”, “2/57”...) I suggest you put your name on each sheet.

E6: Give a formal definition of b_n , the n^{th} **Bell number**, defining any terms you need.

Compute, $b_4 =$
.....

Showing the details, in complete grammatical sentences, derive the EGF of \vec{b} .

End of Class-E

E5: _____ 100pts

E6: _____ 30pts

Total: _____ 130pts

HONOR CODE: “I have neither requested nor received help on this exam other than from my professor (or his colleague).”
Name/Signature/Ord

.....