



Staple!

Ord: _____

Differential Eqns
MAP2302

D-Practice

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Touch: 17Nov2017**Hi.** Whatever you do, **Don't Panic!** This freebie practice is light-years longer than the actual exam.Write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed. Write expressions unambiguously e.g., “ $1/a + b$ ” should be bracketed either $[1/a] + b$ or $1/[a + b]$. (Be careful with negative signs!)Do **not** approx.: If your result is “ $\sin(\sqrt{\pi})$ ” then write that rather than $.9797\cdots$.Use “ $f(x)$ notation” when writing fncs; in particular, for trig and log fncs. E.g., write “ $\sin(x)$ ” rather than the horrible $\sin x$ or $[\sin x]$.**Notation.** For the Laplace transform of f , use $\mathcal{L}(f) = \hat{f}$. Use \mathcal{L}^{-1} for the inVerse Laplace-transform operator. Use **H()** for the **Heaviside fnc**, also called the unit-step fnc. And use **δ()** for the **Dirac delta “function”**.**D1:** Show no work.**a** With **1()** the constant-1 fnc and $F(x) := \sin(5x)$, then, convolution

$$[\mathbf{1} * F](x) = \text{_____}$$

b With **1()** the constant-1 fnc and $F(x) := e^{2x}$, then, convolution

$$[\mathbf{1}^{*4} * F](x) = \text{_____}$$

c With $f(x) := e^{7x}$ and $g(x) := e^{4x}$, then

$$[f * g](5) = \text{_____}$$

d Matrices A, B, U are 2×2 , with U is invertible. Then $e^{A+B} = e^A \cdot e^B$:

$$Ue^B U^{-1} = e^{UBU^{-1}} : \quad AT \quad AF \quad Nei$$

If e^B invertible, then B is invertible: $AT \quad AF \quad Nei$ **e** Fncs $x(t)$ and $y(t)$ satisfy this system of DEs,

$$\begin{aligned} x' + x - 3y &= 0, \\ y' + 6x - 8y &= 0. \end{aligned}$$

It can be written as $\mathbf{Y}' = \mathbf{M} \cdot \mathbf{Y}$,
where $\mathbf{Y} := \begin{bmatrix} x \\ y \end{bmatrix}$ and \mathbf{M} is matrixCharacteristic poly of \mathbf{M} is $\varphi_{\mathbf{M}}(z) =$ A soln has $x(t)$ a linear combination of $e^{\alpha t}$ and $e^{\beta t}$ for numbers $\alpha =$ _____ and $\beta =$ _____.**f** Matrix $G := \begin{bmatrix} 2 & -1 & 3 \\ 4 & -2 & 4 \\ 0 & 0 & 0 \end{bmatrix}$ is nilpotent. Computing, $G^2 =$ The $(1, 3)$ -entry of e^{Gt} is**g** We can re-write function

$$f(t) := \cdot \cos\left(\frac{3}{4}\pi + 5t\right) + \sqrt{2} \cdot \cos\left(\frac{3}{2}\pi + 5t\right)$$

as $f(t) = R \cdot \cos(\theta + 5t)$, for real numbers

$$R = \text{_____} \geq 0 \text{ and } \theta = \text{_____} \in [0, 2\pi).$$

h Let $B := \begin{bmatrix} -1 & 0 \\ 0 & 3 \end{bmatrix}$, $M := \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$. and $R := MBM^{-1}$. Then

$$e^{Rt} = \text{_____}$$

i The Laplace transform of fnc $f(t) := \cos(7t)$ is $\hat{f}(s) =$ For IVP $3y'' - y = \cos(7t)$ with $y(0)=2$ and $y'(0)=5$, then,

$$\hat{y}(s) =$$

$$\mathcal{L}(t^{26}e^{3t})(s) = \text{_____}$$

$$\mathcal{L}(\sin(2t) \cdot \exp(3t))(s) = \text{_____}$$

Determine the inverse-transform, please.

$$\mathcal{L}^{-1}\left(\frac{3s+5}{s^2+2s+5}\right)(t) = \text{_____}$$

k Suppose $y(0) = 2$, $y'(0) = 3$, $y''(0) = 5$. Then $\mathcal{L}(y^{(3)} + 2y')(s)$ equals $[[p(s) \cdot \hat{y}(s)] + q(s)]$ for **polynomials**

$p(s) =$
 and $q(s) =$

OYOP: In grammatical English *sentences*, write your essay on every *third* line (usually), so that I can easily write between the lines.

D2: i Start your essay with this sentence-fragment, and complete the defn using as many sentences as you need:

Saying that function $f: [0, \infty) \rightarrow \mathbb{R}$ has “exponential order 5”, i.e. $f \in \text{Ord}(5)$, means that...

ii Give one example of a continuous fnc $g: [0, \infty) \rightarrow \mathbb{R}$ which lies neither in $\text{Ord}(1)$, nor in $\text{Ord}(2)$, nor in $\text{Ord}(3)$ I.e, this fnc g violates *every* exponential order.

D3: Give a *careful* argument, that $\mathcal{L}(f \circledast g)$ equals $\widehat{f} \cdot \widehat{g}$.

D4: Let $g(t) := \mathbf{H}(t - 5) \cdot f(t - 5)$. Give a *careful* argument, that $\widehat{g}(s)$ equals $e^{-5s} \cdot \widehat{f}(s)$.

D5: i Start your essay with this sentence-fragment, and complete the defn using as many sentences as you need:

An $N \times N$ matrix B is nilpotent if... Moreover, saying that its nilpotency degree is 4 means that...

ii Give an example of 3×3 matrix which has nilpotency degree 2.

End of D-Practice