Topo 2: MTG4303 *and* MTG5317 | **Home-D** | Prof. JLF King 6Sep2015

"*Starting the Semester in Style*"

General. (This is the first exam of 2^{nd}-semester Topology.) Your exam must be typed, but mathematical symbols may be hand-written. Diagrams may be hand-drawn. **Due:** 19Feb at 5PM, slid under my office door. (Home-D given out **11 Feb2008**.)

Notation. TS, topological space. If $\mathcal{A}$ is a collection of subsets of a set Ω, then an "$\mathcal{A}$-cover" (of Ω) is a collection $\mathcal{C} \subset \mathcal{A}$ such that $\bigcup(\mathcal{C}) = \Omega$. □

D0: Given a TS $(\Omega, \mathcal{T})$ and subcollection $\mathcal{A} \subset \mathcal{T}$, suppose that each $\mathcal{A}$-cover has a countable subcover.

a) Prove: If $\mathcal{A}$ is a *base* for $\mathcal{T}$, then Ω is Lindelöf.

b) In contrast, produce an example where $\mathcal{A}$ is a *pre-base* for $\mathcal{T}$, yet Ω is not Lindelöf.

D1: Suppose X is a compact TS and Y is Lindelöf. Prove that $X \times Y$ is Lindelöf.

D2: Munkres, #7 P.143. Let $\mathcal{S}$ denote the subspace topology on Z.

Let $\widehat{Z}$ be the same set, $[\mathbb{R}\times\{0\}] \cup [\{0\}\times\mathbb{R}]$, but equipped with the quotient topology, $\mathcal{Q}$.

b) (cont.) Show that $\widehat{Z}$ has the T_1 separation property.

c) Is one of $\mathcal{S}, \mathcal{Q}$ (strictly) finer/coarser than the other? Compare $\widehat{Z}$ with the Deck of Cards (DoC) space. For y real, let P_y denote the point $(0, y)$ in the plane and let I denote the subset $(0, 1]$ (a half-open interval) of the x-axis.

Let $C_y := \{P_y\} \cup I$. What is the closure of C_y in $\widehat{Z}$? Is C_y open? compact?

D3: Munkres, #10 P.158.

D4: Invent a good problem involving compactness. Now (preferably) solve this problem.

Comments. Recall $\mathbb{R}_\ell$, the reals equipped with the lower-limit topology. We showed in class that $\mathbb{R}_\ell$ *is* Lindelöf, and we discussed $\mathbb{R}_\ell \times \mathbb{R}_\ell$. Recall the Tube Lemma. □

End of Home-D