Linear Algebra
MAS3114 3164

Home-D

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Howdy. Exam is due by **4PM, Friday, 27April**, slid **completely under** my office door, Little Hall 402.

Please write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

D1: Show no work.

a The seq. $\vec{\mathbf{g}} := (g_n)_{n=-\infty}^{\infty}$ is defined by recurrence

$$g_{n+2} = 5g_{n+1} + {-3}g_n$$

and initial conditions $g_0 := -1$ and $g_1 := 2$. So its n^{th} term is $g_n = C_1 \cdot \mu^n + C_2 \cdot \nu^n$, where $\mu<\nu$ are real, and

$C_1=$ ______, $\mu=$ ______,

$C_2=$ ______ and $\nu=$ ______.

[*Hint:* The corresponding matrix is $\mathsf{G} := \begin{bmatrix} 5 & -3 \\ 1 & 0 \end{bmatrix}$. And μ, ν are its eigenvalues.]

b Let A be the 3×3 symmetric real matrix of real quadratic-form

$$7\,{x_1}^2 + 16\,x_1\,x_2 + 3\,{x_2}^2 - 16\,x_2\,x_3 - {x_3}^2\,.$$

Its eigenvalues are -9, 3, 15. Compute an orthogonal matrix U whose change-of-var $\mathbf{x} = \mathsf{U}\mathbf{y}$ transforms $\mathbf{x}^T\mathsf{A}\mathbf{x}$ into a quadratic-form with no mixed term.

$$\mathsf{U} := \left[\quad\middle|\quad\middle|\quad\right].$$

And the new quadratic-form is

$Q(\mathbf{y}) :=$ ______.

Essay questions: On your own sheets of paper, type solns using complete sentences, explaining about HOW you solved each problem. Each essay starts a new page.

D2: Let $\mathsf{M} := \begin{bmatrix} -2 & -6 & -1 & 5 & 9 \\ 3 & 9 & 1 & -7 & -16 \end{bmatrix}$. Then rref(M) is (SNW!)

$$\mathsf{R} := \left[\quad\middle|\quad\middle|\quad\middle|\quad\middle|\quad\right].$$

I For subspace $\mathbf{V} := \text{Nul}(\mathsf{M})$, use back-substitution, and scaling, to produce an *integer basis*

$\mathbf{v}_1:=$ ______, $\mathbf{v}_2:=$ ______,

$\mathbf{v}_3:=$ ______. (SNW!)

II **Show** the steps of Gram-Schmidt to compute an orthogonal *integer basis* for $\mathbf{V}$:

$\mathbf{b}_1:=$ ______, $\mathbf{b}_2:=$ ______,

$\mathbf{b}_3:=$ ______.

Arrange that the Gcd of the entries in each vector is 1, and that the first non-zero value is positive. (Do not show computation of inner-products.)

D3: Observe that $\mathsf{K} := \frac{1}{25}\begin{bmatrix} 39 & -48\,i & -15 \\ 48\,i & 11 & 20\,i \\ -15 & -20\,i & -25 \end{bmatrix}$ is self-adjoint. Its char-poly, $g(x) = x^3 - x^2 - 6x$, is necessarily real. Its (nec. real) evals $\alpha_1 \leqslant \alpha_2 \leqslant \alpha_3$ are

$\alpha_1=$ ______, $\alpha_2=$ ______, $\alpha_3=$ ______.

i Show the spectral decomp $\mathsf{K} = \sum_{j=1}^{3} \alpha_j \mathbf{u}_j \mathbf{u}_j^*$. I.e

$\mathbf{u}_1= [\quad]$, $\mathbf{u}_2= [\quad]$, $\mathbf{u}_3= [\quad]$.

(Put a multiplier in front of each bracket, so as to avoid fractions inside the brackets.)

ii Compute the 3×3 matrix $\mathsf{P}_1 := \mathbf{u}_1\mathbf{u}_1^*$. Give a convincing argument that its LH-action **is** indeed ortho-projection on $\text{Spn}(\mathbf{u}_1)$.

D1:	___ ___	85pts
D2:	___ ___ ___	145pts
D3:	___ ___ ___	145pts
Total:	___ ___ ___	375pts

Print name ______ Ord:

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: ______