

Essay questions with Fill-in-the-blanks.

Please type *–in complete grammatical sentences– solutions* (not just answers) to these problems. Type every 2nd or 3rd line. (Don't Scrunch!) Exam is due **by noon, Friday, 23Apr2004**.

D1: UFs $x(t)$ and $y(t)$ satisfy differential system

$$\text{E1a: } x' - 6x + 3y = 8e^t,$$

$$\text{E1b: } -2x + y' - y = 4e^t,$$

with *initial conditions*

$$\text{E2: } x(0) = -1 \quad \text{and} \quad y(0) = 0.$$

Solve this system using the Laplace xform. Here, use X and Y to denote the xforms of x and y . Write $g(t) := 8e^t$ and $h(t) := 4e^t$, with G and H their xforms. Use the following ideas to solve the system.

Step 1. Compute the xforms of (E1a,b), using (E2). (Justify your steps with the appropriate theorems.) This will give two algebraic eqns, each ITOF X and Y .

Step 2. Solve the algebraic system to get two xform formulae

$$X(s) = \underline{\hspace{10em}},$$

$$Y(s) = \underline{\hspace{10em}}.$$

Each blank will be a *specific* degree-2 rational function in s .

Step 3. Compute the inverse-xforms

$$x(t) = \underline{\hspace{10em}},$$

$$y(t) = \underline{\hspace{10em}}.$$

(Just use partial fractions and table look-up.) *Show me* that your x and y fulfill (E1a,b) and (E2). You can also solve (E1a,b) by the “high school” method shown in class. I don't want to see it, but **you** can use it to know in advance what $x()$ and $y()$ you are shooting for.

D2: For $K = 0, 1, \dots$, let $p_K(z) := z^K$. These are our **power fncs**. Use $\mathbf{h} := p_0$ for the *constant-one* fnc. Inductively derive a formula for the convolution-power

$$\mathbf{h}^{\otimes [K+1]} = \underline{\hspace{10em}}$$

ITOf a power-fnc and factorial. Now write each p_K ITOF convolution-power, then use the associativity of convolution to compute a formula for

$$\dagger: p_K \otimes p_L = \underline{\hspace{10em}}$$

for arbitrary natnums K and L . Your answer will be ITOF factorials and some one power-fnc. Look up “binomial coefficient” and re-express (\dagger) with (most? all?) of the factorials replaced by a binomial coeff. [Hint: Start with the $L=0$ case; then handle $L=1$. You'll see the pattern.]

Look up “multinomial coefficient”. For natnums N, K_0, K_1, \dots, K_N , let $S := \sum_{j=0}^N K_j$. Again use convolution-powers of \mathbf{h} to produce a neat formula: *The convo-product* $p_{K_0} \otimes p_{K_1} \otimes \dots \otimes p_{K_N}$ *equals*

$$\dagger: \underline{\hspace{10em}}.$$

It will use *one* power-fnc and some factorials. Now (in your essay) re-write the formula with a multinomial coeff replacing most of the factorials.

D3: Create an *interesting* Laplace xform problem, or physics DE problem, then solve it nicely.

D1: _____ 155pts

D2: _____ 95pts

D3: _____ 35pts

Total: _____ 285pts

Print
name

Ord:

HONOR CODE: “I have neither requested nor received help on this exam other than from my professor.”

Signature:
