Staple!

Team: ____

Abstract Algebra
MAS4301 3175

Home-D

Prof. JLF King
Tuesday, 08Apr

Hi. Exam is due **4PM, Tuesday, 15Apr2008**, slid under my office door, LIT402. Please write **DNE** in a blank *if* the described object does not exist or if the indicated operation cannot be performed.. Fill-in *all* blanks (*handwriting; don't bother to type*) on this sheet **including** the blanks for the essays!

D1: Show no work.

___ ___ **a** Let $\mathbf{\Gamma}$ be the TTT-aut group of 4×4 TicTacToe, and let $\mathbb{T}$ be the set of 10 TTTs on which $\mathbf{\Gamma}$ acts. Let U denote the upper (horizontal) TTT and let L denote the upper-left to lower-right TTT. The Orb-Stab says

$|\mathrm{Stab}_{\mathbf{\Gamma}}(U)| =$ ______ and $|\mathrm{Stab}_{\mathbf{\Gamma}}(L)| =$ ______.

List $\mathrm{Stab}_{\mathbf{\Gamma}}(U)$ [T, S, F, R]: ______.

___ ___ **b** Perms $\alpha, \beta \in \mathbb{S}_6$ have sigs $\lceil 2^1, 1^4 \rfloor$ and $\lceil 3^2 \rfloor$, resp.. So $|\mathcal{C}(\alpha)| =$ ______, and $|\mathcal{C}(\beta)| =$ ______.

___ ___ **c** Set $G := (\mathbb{Z}_{24}, +)$ and $H := \langle 8 \rangle_G$. In the quotient group $Q := \frac{G}{H}$, then, $\mathrm{Ord}_Q(14 + H) =$ ______.

___ ___ **d** #**26**[P]*191*. $G/H \cong$ Circle: $\mathbb{Z}_2 \times \mathbb{Z}_2$ $\quad \mathbb{Z}_4$ $\quad$ DNE .

___ ___ **e** #**10**[P]*255*. In $\mathbb{Z}_7$: ______, ______, ______, ______.

Essay questions: **Fill-in all blanks.**

D2: Suppose $\mathrm{Z}(G)$ is finite and odd, where G is a (possibly infinite) group. Prove that G has *no* order-2 *normal* subgroup.

D3: A ***PB*** (pegboard) is $\mathbf{B}_{\vec{L}} := [0 \,..\, L_1) \times \ldots \times [0 \,..\, L_{\mathfrak{D}})$ or, more generally, a connected subset $\mathbf{B} \subset \mathbb{Z}^{\times \mathfrak{D}}$. Some cells are occupied by pegs. A jump goes from ●●○ to ○○●, and can be applied in all of the $2\mathfrak{D}$ directions. A ***psn*** (position) is a map $\Lambda \colon \mathbf{B} \to \{\bullet, \circ\}$. Write $\Lambda \curvearrowright \Lambda'$ if a single jump changes Λ to psn Λ'. Write $\Lambda \rightsquigarrow \Lambda'$ if position Λ can go to Λ' in a finite sequence (possibly none) of jumps. Use $\mathbf{P} := \{\text{Set of positions}\}$; so $|\mathbf{P}| = 2^{|\mathbf{B}|}$.

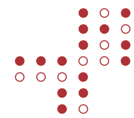
For a cell $\alpha \in \mathbf{B}$, use $\alpha^\bullet$ for the psn with α occupied and all other cells empty. (And α° has α empty, with all other cells occupied. So $\alpha^\bullet, \alpha^\circ \in \mathbf{P}$.) For cells α, β, write $\alpha \ggg \beta$ ("α chevron β") if $\alpha^\circ \rightsquigarrow \beta^\bullet$. Say that α is ***blocked*** (w.r.t $\mathbf{B}$) if there is no cell $\beta \in \mathbf{B}$ with $\alpha \ggg \beta$.

i (Use V for the Klein-4-gp.) Let $\mathcal{K} \colon \mathbb{Z}^{\times \mathfrak{D}} \to V \times \ldots \times V$ ($2^{\mathfrak{D}-1}$ copies) denote the Klein invariant. On board $\mathbb{Z}^{\times \mathfrak{D}}$, there are ______ many $\mathcal{K}$-equiv-classes. Describe them; in dim=2, draw a nice picture of them. Define $\mathcal{K}$ on finite positions by $\mathcal{K}(\Lambda) := \prod_{\alpha \in \Lambda} \mathcal{K}(\alpha)$.

For cells, write $\alpha \overset{\mathcal{K}}{\ggg} \beta$ if $\mathcal{K}(\alpha^\circ) = \mathcal{K}(\beta^\bullet)$. So a cell α is ***$\mathcal{K}$-blocked*** (w.r.t $\mathbf{B}$) if there is *no* β in $\mathbf{B}$ with $\alpha \overset{\mathcal{K}}{\ggg} \beta$. What are the $\mathcal{K}$-blocked cells in $\mathbf{B}_{\vec{L}}$? In ${}^3{+}^7$? In ${}^W{+}^H$? ($\mathbf{B}$ has form "+" of two crossing rectangles.)

ii Is $\ggg$ symmetric?; is $\overset{\mathcal{K}}{\ggg}$? Is $\ggg$ transitive?; is $\overset{\mathcal{K}}{\ggg}$? How do the answers depend on the board $\mathbf{B}$?

iii Call the righthand psn "Blob". The board, $\mathbf{B}$, is enclosed by the decagon of Blob's edges. Prove that Blob is blocked. [*Hint:* Use $\mathcal{K}()$ together with hop-equivalence.] Can Blob be jump-reduced to a single peg on the board, when allowing jumps on *all* of $\mathbb{Z} \times \mathbb{Z}$?



iv On $\mathbf{B} := 4 \times 4$, is $\overset{\mathcal{K}}{\ggg}$ the same as $\ggg$? In $\mathbb{Z}^{\times \mathfrak{D}}$, two cells are ***hop-equivalent*** if a peg could jump from one cell to the other, assuming that a peg to-be-jumped-over materialized when needed. On $\mathbb{Z}^{\times \mathfrak{D}}$ there are ______ many hop-equivalence-classes. How can you combine hop-equivalence and $\mathcal{K}()$ to find if $\alpha \ggg \beta$?

v Generalize everything. E.g, a ***coal***(escence) moves ●○● to ○●○. Not only is $\mathcal{K}()$ jump-invariant, it is also coal-invariant! On $\mathbf{B} := 4 \times 4$, what is $\overset{coal}{\ggg}$? $\underset{jump}{\overset{coal}{\ggg}}$?

vi Draw a Klein $\{a, b, c\}$-labeling of the vertices of a triangular tesselation (equiv., the regions of a hexagonal board). What are the $\mathcal{K}$-blocked cells on T_4? What can you say about T_n? (Triangle with n cells on each edge; has $\frac{[n+1]n}{2}$ cells.)

End of Home-D

D1:	___ ___ ___	125pts
D2:	___ ___	35pts
D3:	___ ___ ___	115pts
Total:	___ ___ ___	275pts

HONOR CODE: *"I have neither requested nor received help on this exam other than from my team-mates and my professor (or his colleague)."* *Name/Signature/Ord*

Ord: ______

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