ACT
MAA4211 7222

Home-D

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Touch: 6May2016

D3: Produce (with proof and pictures) a fnc $H{:}\mathbb{R}{\to}\mathbb{R}$ such that $\mathrm{DisCty}(H) = \mathbb{D} \cup \pi\mathbb{Q}$.

Hello. Essays violate the CHECKLIST at *Grade Peril*!
Exam is due by ~~3:30PM, Thursday, 20Nov2008,~~

10:30AM, Friday, 21Nov2008,

slid completely **under** my office door, LIT402. Please write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.
Fill-in *all* blanks (*handwriting; don't bother to type*).

End of Home-D

D1:	_____	95pts
D2:	_____	115pts
D3:	_____	55pts
Poorly stapled, or missing ordinals :	_____	−5pts
Missing names, or honor sigs :	_____	−5pts
Total:	_____	265pts

D1: Show no work.

a Let $h(x) := [x^2 - 4] \cdot \mathbf{1}_K(x)$ be a map on $\mathbb{R}$, where $K := \{[-1]^n + 1 - \frac{2}{n}\}_{n=1}^{\infty}$. So $\mathrm{DisCty}(h)$ equals

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b Define $\psi{:}(\dot{\mathbb{R}}, \boldsymbol{\sigma}){\to}(\mathbb{R}, |\cdot|)$ by $\psi(x) := \boldsymbol{\sigma}(x, 7)$. Then $\mathrm{Lip}(\psi) =$ . . . .

c The identity map $Id{:}(\mathbb{R}, \boldsymbol{\sigma}){\to}(\mathbb{R}, |\cdot|)$ has Lipschitz constant . . . .

d For $n = 2, 3, 4, \ldots$, let $g_n{:}[0,1]{\to}\mathbb{R}$ be the P.L fnc with these cutpoint and height tuples:

$$\vec{\mathbf{p}} := \big(0, 1/n^3, 1/n^2, 1\big) \quad \text{and}$$
$$\vec{\mathbf{h}} := \big(0, \quad n, \quad 0, \quad 0\big).$$

Circle those senses in which seq $\big(g_n\big)_{n=1}^{\infty}$ converges:
`pointwise` $\quad \|\cdot\|_3 \quad \|\cdot\|_2 \quad \|\cdot\|_1 \quad \|\cdot\|_{\sup}$

Essay questions: For each question, carefully type a triple–spaced essay solving the problem.
Each essay starts a new page.

D2: We consider real numbers $z_n \to 7$ and (possibly discontinuous) fncs $g, f_n{:}\mathbb{R}{\to}[0,6]$ such that $f_n \xrightarrow{\text{ptwise}} g$.

i Produce an example with *continuous* f_n and g, yet $\limsup_n f_n(z_n) = 6$ and $\liminf_n f_n(z_n) = 0$.
[*Hint:* Draw LARGE, careful, colorful pictures of your fncs.]

ii Construct an example with $f_n \xrightarrow{\text{uniformly}} g$, yet $\limsup_n f_n(z_n) = 6$ and $\liminf_n f_n(z_n) = 0$.
[*Hint:* Probably some of the fncs are *dis*continuous. Draw LARGE, colorful, careful pictures.]

iii Suppose each f_n is continuous, **and** $f_n \xrightarrow{\text{uniformly}} g$. Prove that $\lim_n f_n(z_n) = g(7)$.

Staple!