Differential Eqns MAP2302 3145 **D-home-Alt** Prof. JLF King *Touch*: 17Oct2017

Essay questions with Fill-in-the-blanks. Please type –in *complete grammatical sentences*– **solutions** (not just answers) to these problems. Type every 2nd or 3rd line. (Don't Scrunch!) Exam is due **by noon, Friday, 23Apr2004**.

D1: UFs $x(t)$ and $y(t)$ satisfy differential system

E1a: $x' - 6x + 3y = 8e^t$,

E1b: $-2x + y' - y = 4e^t$,

with *initial conditions*

E2: $x(0) = -1$ and $y(0) = 0$.

Solve this system using the Laplace xform. Here, use X and Y to denote the xforms of x and y. Write $g(t) := 8e^t$ and $h(t) := 4e^t$, with G and H their xforms. Use the following ideas to solve the system.

Step 1. Compute the xforms of (E1a,b), using (E2). (Justify your steps with the appropriate theorems.) This will give two algebraic eqns, each ITOf X and Y.

Step 2. Solve the algebraic system to get two xform formulae $X = \frac{B}{C}$ and $Y = \frac{Q}{R}$, where
$B(s)=$ ________, $C(s)=$ ________,

$Q(s)=$ ________ and $R(s)=$ ________

are **polynomials**. Further, $B \perp C$ and $Q \perp R$.

Step 3. Compute the inverse-xforms

$x(t) =$ ________________,

$y(t) =$ ________________.

(Just use partial fractions and table look-up.) *Show me* that your x and y fulfill (E1a,b) and (E2). You can also solve (E1ab,E2) by the "high school" method shown in class. I don't want to see it, but **you** can use it to know in advance what $x()$ and $y()$ you are shooting for.

D2: For $K = 0, 1, \ldots$, let $p_K(z) := z^K$. These are our ***power fncs***. Use $\mathbf{h} := p_0$ for the *constant-one* fnc. Inductively derive a formula for the convolution-power

$\mathbf{h}^{\circledast[K+1]} =$ ________________

ITOf a power-fnc and factorial. Now write each p_K ITOf convolution-power, then use the associativity of convolution to compute a formula for

†: $p_K \circledast p_L =$ ________________

for arbitrary natnums K and L. Your answer will be ITOf factorials and some one power-fnc. Look up "binomial coefficient" and re-express (†) with (most? all?) of the factorials replaced by a binomial coeff. [*Hint:* Start with the L=0 case; then handle L=1. You'll see the pattern.]

Look up "multinomial coefficient". For natnums $N, K_0, K_1, \cdots, K_N$, let $S := \sum_{j=0}^{N} K_j$. Again use convo-powers of $\mathbf{h}$ to produce a neat formula: *The convo-product* $p_{K_0} \circledast p_{K_1} \circledast \cdots \circledast p_{K_N}$ *equals*

‡: ________________.

It will use *one* power-fnc and some factorials. Now (in your essay) re-write the formula with a multinomial coeff replacing most of the factorials.

D3: Create an *interesting* Laplace xform problem, or physics DE problem, then solve it nicely.

D1:	___ ___ ___	155pts
D2:	___ ___	95pts
D3:	___ ___	35pts
Total:	___ ___ ___	285pts

Print name ________________ Ord:

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: ________________