Sets and Logic
MHF3202 4628

Class-D

Prof. JLF King
Friday, 04Apr2008

D1: ___ ___ ___ 210pts

D2: ___ ___ 65pts

Total: ___ ___ ___ 275pts

Print name ______________________ Ord:

D1: Show no work. Write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

a) Mathematician X wrote (in German) to Dedekind: *"Can a surface . . . be uniquely referred to a line . . . so that for every point on the surface there is a corresponding point of the line and, conversely, for every point of the line there is a corresponding point of the surface? I think that answering this question would be no easy job, despite the fact that the answer seems so clearly to be "no" that proof appears almost unnecessary."*

Yet three years later, when he proved that the unit-square *can* be mapped bijectively to the unit interval, he wrote *"I see it, but I don't believe it!"* Mathematician X is: Circle: *Archimedes Cantor Cohen* DNE *Euler Gödel Machen.*

X worked primarily during: *1600s 1700s 1800s 1900s.*

b) An explicit bijection $F:\mathbb{N}\hookrightarrow\!\!\!\!\to\mathbb{Z}$ is this:

When n is *even*, then $F(n) :=$ ______________.

When n is *odd*, then $F(n) :=$ ______________.

c) The map $f(k,n) := 2^k \cdot [1+2n]$ is a bijection from $\mathbb{N}\times\mathbb{N}\hookrightarrow\!\!\!\!\to\mathbb{Z}_+$. And $f^{-1}(176) = ($ ________ , ________ $)$.

d) $\mathcal{P}(\mathcal{P}(\text{3-stooges}))$ has ______________ many elements.

e) To the interval $J := (-\frac{\pi}{2}, \frac{\pi}{2})$, define a bijection $g:(0,1)\hookrightarrow\!\!\!\!\to J$ by $g(x) :=$ ______________.

Using this g and a trigonometric fnc, define a bijection $h:(0,1)\hookrightarrow\!\!\!\!\to\mathbb{R}$ by $h(x) :=$ ______________

f) Repeating decimal $2.\overline{75}$ equals $\frac{n}{d}$, where posints $n \perp d$ are $n=$ ______________ and $d=$ ______________.

g) Mod $K{:=}47$, the recipr. $\langle\frac{1}{21}\rangle_K =$ ________ $\in[0\,..\,K)$. [*Hint:* ↯] So $x=$ ________ $\in[0\,..\,K)$ solves $4 - 21x \equiv_K 2$.

Essay question: Carefully write a triple–spaced, grammatical, essay solving the problem.

D2: Let $L(n) := [n^3 - n]$, where $L:\mathbb{Z}\to\mathbb{Z}$. By induction on n, prove that $\forall n\in\mathbb{N}:\ \ 3 \bullet\!\!\mid L(n)$.

Explicitly prove the base case. Explicitly state the induction *implication*, then prove that it holds for each $n\in\mathbb{N}$.

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: ______________________