Staple!

Ord: ___ ___

Linear Algebra
MAS4105 *6137*

Class-D

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Wednesday, 16Mar2016

D1: Show no work. Please write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

___ ___ **a** Let $\mathscr{L}$ be this list of 8 symbols: $A, G, R, T, U, \alpha, \beta, \omega$.

In matrix-eqn $\begin{bmatrix} R & U & G \\ & A & \\ T & & \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \alpha \\ \beta \\ \omega \end{bmatrix}$, Cramer's Rule writes x_3 as ratio $f(\mathscr{L}) \big/ q(\mathscr{L})$ of polynomials

$f(\mathscr{L}) =$ ______________________________

and $q(\mathscr{L}) =$ ______________________________.

___ ___ **b** Let $\mathsf{S}:\mathbb{R}^2 \circlearrowleft$ rotate the plane CCW by $60°$, then vertically stretch by a factor of 8. W.r.t the std basis,

$$[\![\mathsf{S}]\!]_{\mathcal{E}}^{\mathcal{E}} = \left[\begin{array}{c|c} & \\ \hline & \end{array}\right].$$

___ ___ **c** Every $3{\times}3$ matrix M has $\operatorname{Det}(5\mathsf{M}) = \alpha \cdot \operatorname{Det}(\mathsf{M})$, where α is [circle]

5^9 5^3 5^2 5 3^5 9^5 3^{25} None-of-these

___ ___ **d** The seq. $\vec{\mathbf{g}} := (g_n)_{n=-\infty}^{\infty}$ is defined by recurrence

$$g_{n+2} = 4g_{n+1} + 5g_n$$

and initial conditions $g_0 := 11$ and $g_1 := 7$. So its n^{th} term is $g_n = C_1 \cdot \mu^n + C_2 \cdot \nu^n$, where $\mu \leqslant \nu$ are real, and

$C_1 =$ ______________, $\mu =$ ______________,

$C_2 =$ ______________ and $\nu =$ ______________.

___ ___ **e** Let $\mathsf{M}(x) := \begin{bmatrix} 3x-2 & 7x^4-8 & 10 \\ 5 & 9x-2 & 2x-8 \\ 8x & x^5-2 & x^3+2 \end{bmatrix}$.

The high-order term of polynomial $\operatorname{Det}(\mathsf{M}(x))$ is Cx^N, where $C =$ __________ and $N =$ __________.

___ ___ **f** Let $\mathsf{T}:\mathbb{R}^2 \to \mathbb{R}^2$ by reflecting the plane across the θ-angle line. [Picture on blackboard.] Then $\mathsf{M} := [\![\mathsf{T}]\!]_{\mathcal{E}}^{\mathcal{E}}$ equals

$\begin{bmatrix} \cos(\alpha) & \cos(\beta) \\ \sin(\alpha) & \sin(\beta) \end{bmatrix}$, where $\alpha =$ ______________ and

$\beta =$ ______________. Also, $\operatorname{Det}(\mathsf{M}) =$ __________.

[*Hint:* Where does T send $\mathbf{e}_1$? And where is $\mathsf{T}(\mathbf{e}_2)$?]

g Here, matrices $\mathsf{B}, \mathsf{C}, \mathsf{X}, \mathsf{Y}$ range over $\text{MAT}_{2\times2}(\mathbb{R})$.

1. If 3 is an B-*e*val, then 9 is an B^2-*e*val. *T* *F*
2. Suppose $\mathbf{w} \neq \mathbf{0}$ is an *e*vec for B^2. Then $\mathbf{w}$ is an *e*vec for B. *T* *F*
3. If $\mathsf{B} \overset{\text{sim}}{\sim} \mathsf{C}$ then $[\mathsf{B}^2 + \mathsf{B}^3] \overset{\text{sim}}{\sim} [\mathsf{C}^2 + \mathsf{C}^3]$. *T* *F*
4. If $\mathsf{B} \overset{\text{sim}}{\sim} \mathsf{C}$ and $\mathsf{X} \overset{\text{sim}}{\sim} \mathsf{Y}$ then $\mathsf{BX} \overset{\text{sim}}{\sim} \mathsf{CY}$. *T* *F*

D2: OYOP: Essay: *Write on every* ***third*** *line, so that I can easily write between the lines.*

DEFN: A collection $\mathcal{C} := \{\mathbf{U}_1, \ldots, \mathbf{U}_K\}$ of subspaces is ***linearly-independent*** if: . . .

THM: For linear-transformation $\mathsf{T}:\mathbf{V}\to\mathbf{V}$, eigenspaces $\mathbf{W}_1, \ldots, \mathbf{W}_8$ have (distinct) eigenvalues $\beta_1, \ldots, \beta_8$. Prove that $\mathcal{D} := \{\mathbf{W}_1, \ldots, \mathbf{W}_8\}$ is linearly-independent.

End of Class-D

D1:	___ ___ ___	160pts
D2:	___ ___	65pts
Total:	___ ___ ___	225pts

Please PRINT your name and ordinal. Ta:

______________________________ Ord:

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: ______________________________