Linear Algebra MAS4105 *6137* | **Class-D** | Prof. JLF King 31Aug2015

OYOP: For the 2 Essays: *Write your grammatical English **sentences** on every **third** line, so that I can easily write between the lines.*

For a square matrix M, the ***"minimum polynomial** of M"* [***minpoly***] is the *smallest degree* monic polynomial $\Upsilon_M()$ st. $\Upsilon_M(M)$ is the zero-matrix. It is always a polynomial-divisor of the charpoly of M.

D1: Use $\langle\cdot,\cdot\rangle$ for an inner-product on $\mathbb{R}$-vectorspace $\mathbf{V}$.

x1 State the Cauchy-Schwarz Inequality Thm, carefully stating the IFF-condition for equality. x2 **Prove** the C-S Inequality Thm, using the axioms for inner-product.

D2: Suppose S and T are 3×3 matrices, with S *invertible*. Let $f()$ be the charpoly of product ST. Let $g()$ be the charpoly of TS. *Carefully prove* that $f = g$.

D3: *Short answer:* Show no work.

Please write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

__ __ a A multivariate polynomial, where each monomial has the same degree, is circle

monogamous	**atrocious**	**gregarious**
homogeneous	**expialadocious**	**monic**
manic **unitary**	**Unitarian**	**utilitarian**

__ __ b Blanks$\in\mathbb{R}$. So $\frac{1}{3+2i}=$ ______ $+i\cdot$ [______].

And $\dfrac{5-i}{3+2i} =$ ______ $+i\cdot$ [______].

By the way, $|5-3i| =$ ______.

__ __ c Let $A := \left[\begin{smallmatrix}1&0\\0&0\end{smallmatrix}\right]$ and $B := \left[\begin{smallmatrix}0&1\\0&0\end{smallmatrix}\right]$. Thus

$\Upsilon_{AB}(x)=$ ______ ; $\Upsilon_{BA}(x)=$ ______.

__ __ d In $\mathbb{R}^3$, let $\mathbf{v} := (1,\text{-}1,4)$. and $\mathbf{W} := \text{Spn}(\{\mathbf{v}\})$. Solve equation $\langle\mathbf{v},\mathbf{u}\rangle = 0$ for $\mathbf{u}$, to get an *integer-basis*

$\mathbf{u}_1$=(__ , __ , __) and $\mathbf{u}_2$=(__ , __ , __) for $\mathbf{W}^\perp$.

Gram-Schmidtify to obtain an *orthogonal* integer-basis

$\mathbf{y}_1$=(__ , __ , __) and $\mathbf{y}_2$=(__ , __ , __).

[DYCheck that $\mathbf{y}_1 \perp \mathbf{v}$, $\mathbf{y}_2 \perp \mathbf{v}$, $\mathbf{y}_1 \perp \mathbf{y}_2$? –with both $\mathbf{y}_i \neq \mathbf{0}$.]

e $\mu=$ ______ $\leqslant \nu =$ ______

are the eigenvalues of $G := \begin{bmatrix}2&3\\4&1\end{bmatrix}$. Let $D := \begin{bmatrix}\mu&0\\0&\nu\end{bmatrix}$. Then $D = U^{-1}GU$ where the 2×2 *integer* matrix U is

$$U = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}.$$

End of Class-D

D1:	__ __	90pts
D2:	__ __	45pts
D3:	__ __ __	120pts
Total:	__ __ __	255pts

Please PRINT your name and ordinal. Ta:

Ord: ______

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: ______