Differential Eqns
MAP2302

D-Class

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Write **DNE** in a blank i̲f̲ the described object does not exist or if the indicated operation cannot be performed. **Write expressions unambiguously e.g, "$1/a + b$" should be bracketed either $[1/a] + b$ or $1/[a+b]$. (Be careful with negative signs!)**

Do **not** approx.: If your result is "$\sin(\sqrt{\pi})$" then write that rather than $.9797\cdots$.

Use "$f(x)$ **notation**" when writing fncs; in partic- ular, for trig and log fncs. E.g, write "$\sin(x)$" rather than the horrible $\sin x$ **or** $[\sin x]$.

For the *Laplace transform* of f, use $\mathcal{L}(f) = \widehat{f}$.

D1: Show no work.

a With $f(x) := x^2$ and $g(x) := e^{3x}$, then $\big[f \circledast g\big](t) =$ $\underset{\llcorner\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\lrcorner}{}$.

b Fncs $x(t)$ and $y(t)$ satisfy this system of DEs,

$$x' + 8x + 6y = 0,$$
$$y' - 9x - 7y = 0.$$

It can be written as $\mathsf{Y}' = \mathsf{M}{\cdot}\mathsf{Y}$,

where $\mathsf{Y} := \big[\begin{smallmatrix}x\\y\end{smallmatrix}\big]$ and M is matrix $\underset{\llcorner\cdots\cdots\cdots\cdots\lrcorner}{}$.

Characteristic poly of M is $\wp_{\mathsf{M}}(z) =$ $\underset{\llcorner\cdots\cdots\cdots\cdots\lrcorner}{}$

A soln has $x(t)$ a linear combination of $e^{\alpha t}$ and $e^{\beta t}$ for *numbers* $\alpha = \underset{\llcorner\cdots\cdots\cdots\lrcorner}{}$ and $\beta = \underset{\llcorner\cdots\cdots\cdots\lrcorner}{}$.

c Matrices $\mathsf{U}, \mathsf{G}, \mathsf{R}$ are 3×3, with U invertible and R nilpotent. [Use $\mathbf{I}$ for the 3×3 identity matrix.]

Matrix $\mathsf{U}\mathsf{R}\mathsf{U}^{-1}$ *is nilpotent*:   *AT*   *AF*   *Nei*

Each entry of $e^{t\mathsf{R}}$ *is a polynomial*:   *AT*   *AF*   *Nei*

Matrix e^{R} *is nilpotent*:   *AT*   *AF*   *Nei*

R^2 *is the zero-matrix*:   *AT*   *AF*   *Nei*

Matrix $e^{[\mathsf{G}+\mathbf{I}]\mathsf{G}}$ *equals* $e^{\mathsf{G}} \cdot e^{\mathsf{G}^2}$:   *AT*   *AF*   *Nei*

Matrix $e^{[\mathsf{G}^2]}$ *equals* $\big[e^{\mathsf{G}}\big]^2$:   *AT*   *AF*   *Nei*

d Let $\mathsf{A} := \big[\begin{smallmatrix}2 & 0\\0 & -1\end{smallmatrix}\big]$, $\mathsf{M} := \big[\begin{smallmatrix}-1 & 2\\-1 & 3\end{smallmatrix}\big]$ and $\mathsf{R} := \mathsf{M}\mathsf{A}\mathsf{M}^{-1}$. Then the $(2,2)$ entry of $e^{\mathsf{R}t}$ is $\underset{\llcorner\cdots\cdots\cdots\cdots\cdots\lrcorner}{}$.

e Suppose $y(0) = -2$, $y'(0) = 5$, $y''(0) = 2$. Then $\mathcal{L}\big(y^{(3)} + y^{(2)} + 3y\big)(s)$ equals $\big[[B(s){\cdot}\widehat{y}(s)] + C(s)\big]$ for **polynomials**

$C(s) = \underset{\llcorner\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\lrcorner}{}$

and $B(s) = \underset{\llcorner\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\lrcorner}{}$.

OYOP: *In grammatical English* sentences*, write your essay on every* third *line* (usually)*, so that I can easily write between the lines.*

D2: Give a *careful* argument that $\mathcal{L}(f \circledast g)$ equals $\widehat{f} \cdot \widehat{g}$.

End of D-Class

D1:	___ ___ ___	125pts	
D2:	___ ___	70pts	
Total:	___ ___ ___	195pts	

Please PRINT your name *and* ordinal*. Ta:*

Ord:
$\llcorner\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\lrcorner$

Staple!