Advanced Calc
MAA4102 & MAA5104

Class-D

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Hello. Abbrevs: **WtSaCi** for "Write the Sentence and Complete it". **G.E.O** for "Give (an) example of". **ITOf** for "in terms of". **st.** for "such that"

Use **nv-** for "non-void", e.g "consider a nv-closed set K". Use **MS** for "metric space". Use **RI** for "Riemann Integrable" or "Riemann Integral".

Use $\overline{\mathbb{R}}$ for $[-\infty,+\infty]$, the "extended reals".

For each of the limit questions, write "$+\infty$", "$-\infty$", a real number, or *–if none of these–* "DNE". In general Please write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

D1: Show no work.

a Suppose $H:\mathbb{R}\to\mathbb{R}$ is cts. Then H is *uniformly continuous.* AT AF Nei

If $F:\mathbb{R}\circlearrowleft$ is diff'able, then F' is cts. AT AF Nei

If $F:\mathbb{R}\circlearrowleft$ is diff'able, then F is cts. AT AF Nei

b Suppose unif-cts $h:\mathbb{R}\to\mathbb{R}$ is diff'able. Then $|h'|$ is uniformly-bounded. AT AF Nei

Suppose diff'able $f:[-3,3]\to\mathbb{R}$ has $|f'|\leqslant 22$. Then f' is cts on $[-3,3]$. AT AF Nei

c $\dfrac{\mathrm{d}}{\mathrm{d}x}\displaystyle\int_5^{\sin(x)}\sin(1/t)\,\mathrm{d}t=$ ⌞ . . . . . ⌟.

d Let $F(x):=\displaystyle\int_{x+7}^{x^3}\cos\big(\cos(t)\big)\,\mathrm{d}t$. Consequently

$\dfrac{\mathrm{d}F}{\mathrm{d}x}=$ ⌞ . . . . . ⌟.

e $\displaystyle\lim_{x\searrow 0}[2+5x]^{1/x}=$ ⌞ . . . . . ⌟.

$\displaystyle\lim_{z\searrow 0}\frac{\sin(z)-z}{z^3}=$ ⌞ . . . . . ⌟.

f $\int_2^7 x\cdot 3^x\,\mathrm{d}x=$ ⌞ . . . . . ⌟.

Note: IBParts. You may use $K:=1/\log(3)$ in your answer. An antideriv of 3^x is $K\cdot 3^x$.

g Let $H(x):=\cos(7x)$. Its fifth *Taylor polynomial* is $\sum_{n=0}^{4}B_n x^n$, where $B_0=$ ⌞ . . . ⌟, $B_1=$ ⌞ . . . ⌟, $B_2=$ ⌞ . . . ⌟, $B_3=$ ⌞ . . . ⌟ and $B_4=$ ⌞ . . . ⌟.

[*N.B* You may write $7^2, 7^3$ etc without multiplying out.]

h Define seq $\vec{\mathbf{b}}$ by $b_n:=\frac{1}{n\cdot[n+1]}$. Get a closed-formula for $\displaystyle\sum_{j=8}^{2006}b_j=$ ⌞ . . . . . ⌟. [*Hint:* Express your answer as a difference of rationals. Can you view $\vec{\mathbf{b}}$ as the discrete deriv of . . . ?]

D2: α G.E.O a *differentiable* fnc f on $\mathbb{R}$, with f' **not** cts at 7:

$$f(x):=\begin{cases}\llcorner\ldots\lrcorner & \text{if } \llcorner\ldots\lrcorner\\ \llcorner\ldots\lrcorner & \text{if } \llcorner\ldots\lrcorner\end{cases}.$$

β For $S\subset\mathbb{R}$, use $\mathbf{1}_S$ for the *indicator fnc* of S: Graph $F(x):=4\cdot\mathbf{1}_{(-\infty,7]}(x)+x\cdot\mathbf{1}_{[5,\infty)}(x)$.

Use a *full sheet of paper*, make the graph LARGE and CL.

Essays. On your own sheets of lined paper, give the following definitions or proofs. No "scratch work" accepted, only complete, grammatical, coherent sentences. Write **every 2nd or every 3rd line**.

D3: State the Cauchy Mean Value Theorem.

D4: Let $J:=[3,7]$. **WtSaCi**:

A ***pointed partition*** (pptn) $P=(\vec{x},\vec{Q})$ on J is. . . .

Its ***mesh***, Mesh(P), is the number. . . .

Given $f:J\to\mathbb{R}$, its ***Riemann Sum*** is $\mathrm{RS}_f(P)$ . . . .

A bnded fnc $h:J\to\mathbb{R}$ is ***Riemann integrable*** IFF. . . .

D1:	___ ___ ___	200pts
D2:	___ ___	75pts
D3:	___ ___	65pts
D4:	___ ___	85pts
Total:	___ ___ ___	425pts

Print name ⌞ . . . . . ⌟ Ord:

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: ⌞ . . . . . ⌟