Sets and Logic MHF3202 4628	**Prac-D**	Prof. JLF King Friday, 04Apr2008

D1: Show no work. Write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

a) Repeating decimal $7.45\overline{123}$ equals $\frac{n}{d}$, where posints $n \perp d$ are $n=$ ________ and $d=$ ________.

b) $\mathcal{P}(\mathcal{P}(\{3,4,5,6\}))$ has ________ many elements.

c) To the interval $J := (-\frac{\pi}{2}, \frac{\pi}{2})$, define a bijection $g: (5,6) \hookrightarrow\!\!\!\rightarrow J$ by $g(x) :=$ ________.

Using this g and a trigonometric fnc, define a bijection $h: (5,6) \hookrightarrow\!\!\!\rightarrow \mathbb{R}$ by $h(x) :=$ ________

d) Each three sets Ω, B, C engender a natural bijection, $\Theta: \Omega^{B\times C} \hookrightarrow\!\!\!\rightarrow [\Omega^B]^C$, defined, for each $f \in \Omega^{B\times C}$, by

$\Theta(f) := \Big[c \mapsto [$ ________ $]\Big]$.

Its inverse-map $\Upsilon: [\Omega^B]^C \hookrightarrow\!\!\!\rightarrow \Omega^{B\times C}$ has, for $g \in [\Omega^B]^C$, $\Upsilon(g) := \Big[(b,c) \mapsto [$ ________ $]\Big]$.

e) Mod $K{:=}50$, the recipr. $\langle \frac{1}{21} \rangle_K =$ ________ $\in [0 .. K)$.

[*Hint:* ↯] So $x=$ ________ $\in [0 .. K)$ solves $4 - 21x \equiv_K 1$.

f) LBolt: $\mathrm{GCD}(45, 63) =$ ________ $\cdot 45 +$ ________ $\cdot 63$.

So (LBolt again) $G := \mathrm{GCD}(45, 63, 105) =$ ________ and

________ $\cdot 45 +$ ________ $\cdot 63 +$ ________ $\cdot 105 = G$.

Essay questions: For each question, carefully write a triple–spaced, grammatical, essay solving the problem.

D2: Let $L(n) := \frac{n}{n+1}$. And let $R(n) := \sum_{k=1}^{n} \frac{1}{k \cdot [k+1]}$. By induction on n, prove that $\boxed{\forall n \in \mathbb{N}: \quad L(n) = R(n)}$. *Explicitly* prove the base case. *Explicitly* state the induction implication, then prove that it holds for each $n \in \mathbb{N}$.

D3: Prove that the map $f: \mathbb{N}\times\mathbb{N} \to \mathbb{Z}_+$, with $f(k,n) := 2^k \cdot [1 + 2n]$, is injective. Now prove that it is surjective.

D4: Let $\mathcal{P}_\infty$ denote the collection of all infinite subsets of $\mathbb{N}$. Define a relation $\doteqdot$ on $\mathcal{P}_\infty$ by: $A \doteqdot B$ IFF $A \cap B$ is infinite.

Either prove that $\doteqdot$ *is* transitive, or else produce three explicit sets $A, B, C \in \mathcal{P}_\infty$ showing that $\doteqdot$ is *not* transitive.

D5: Suppose $\mathbf{J}$ is a set, and $\{B_i\}_{i\in\mathbf{J}}$ is a set-of-sets, with each $B_i \subset \mathbb{Z}_+$, and $[i{\neq}k] \Rightarrow B_i {\neq} B_k$. Each distinct index-pair $i, k \in \mathbf{J}$ has $B_i \cap B_k = \varnothing$. Construct, with proof, an *injection* $f: \mathbf{J} \hookrightarrow \mathbb{N}$, to conclude that $\mathbf{J}$ is only countable.