Congruence

Here is an example soln to (B2).

Defn. For integers x and y, and posint K, say that

$$x \equiv_K y$$

IFF $[x-y] \mid\!\bullet\, K$; ie. **there exists** an **integer** n such that

$$x - y = nK. \qquad \square$$

Remark. Henceforth, fix a posint K and let "$\equiv$" mean "$\equiv_K$". □

1: Theorem. *Consider integers b, β, g, γ. If*

$$b \equiv \beta \quad \text{and} \quad g \equiv \gamma$$

then

$$*: \qquad [b \cdot g] \equiv [\beta \cdot \gamma]. \qquad \Diamond$$

Proof. By the above defn of congruence, **there exist** integers ℓ and m **such that**

$$\begin{aligned} b &= \beta + \ell K \quad \text{and} \\ g &= \gamma + mK. \end{aligned}$$

Multiplying the two eqns yields that

$$¥: \quad \begin{aligned} \text{LhS}(*) &= \beta\gamma + \beta{\cdot}mK + \ell K{\cdot}\gamma + \ell K{\cdot}mK \\ &\stackrel{\text{note}}{=\!=} \text{RhS}(*) + nK, \end{aligned}$$

where

$$n := \beta m + \ell\gamma + \ell K m.$$

Hence n is an **integer**, since $\mathbb{Z}$ is sealed under multiplication and addition. Thus (¥) shows that

$$\text{LhS}(*) \equiv \text{RhS}(*),$$

as desired. ♦

Filename: Classwork/NapoSelo/SeLo2009t/congruence-soln.SeLo2009t.latex
As of: *Monday 31Aug2015.* Typeset: *1Sep2016 at 18:49.*