Staple!

Team C ______________________

Linear Algebra
MAS4105 5441

C-Home

Prof. JLF King
Touch: 1Feb2022

Greetings, Humanoid. Your essays violate the CHECKLIST at *Your Peril*! "IP" means "inner product". Exam is due **by 4:30PM, Thursday, 08Dec2005**.

C1: Short answer: Show no work. Write **DNE** if the object does not exist or the operation cannot be performed. NB: DNE $\neq \{\} \neq 0$.

___ ___ **z** Professor King hopes I'll stop by next semester to chat. **True**
He has an office? Will there be chocolate?

___ ___ **a** Compute the $\mathbb{L}_p$-norm of $\mathbf{v} := (2\mathbf{i}, -4, 2, \mathbf{i})$.
$\|\mathbf{v}\|_1=$ ______ , $\|\mathbf{v}\|_2=$ ______ , $\|\mathbf{v}\|_3=$ ______ , $\|\mathbf{v}\|_\infty=$ ______ .

___ ___ **b** #4(b)$^{P.}$336. $\|\mathsf{A}\|=$ ______ . $\langle \mathsf{A}, \mathsf{B}\rangle=$ ______ .

___ ___ **c+** In $\mathbb{C}^3$, let $\mathbf{W} := \mathrm{Spn}((\mathbf{i}, -1, 1))$. As col-vecs, an orthonormal basis for $\mathbf{W}^\perp$ is: ______ ______ .

___ ___ **d** Column-vecs $\left[\begin{smallmatrix}-1\\2\end{smallmatrix}\right]$ and $\left[\begin{smallmatrix}1\\3\end{smallmatrix}\right]$ are orthonormal relative to IP $\langle \mathbf{v}, \mathbf{b}\rangle := \mathbf{v}^{\mathrm{t}}\mathsf{M}\mathbf{b}$ where the real symmetric (positive definite) matrix M is ______ .

___ ___ **e+** In $\mathbb{R}^4$, let $\mathbb{L}_0$ be the line passing through the origin and the point $Q := (1, 2, 3, 4)$. Let $\mathbb{L}_1$ be the line $t \mapsto (2, -1, 0, 1) + t(5, 0, 1, 2)$. The (orthogonal) dist. between lines $\mathbb{L}_0$ and $\mathbb{L}_1$ is ______ .

___ ___ **f+** In $\mathbb{R}^3$, let $\mathbb{L}_3$ be the line $3 - x = \frac{y-6}{2} = z - 3$. Let $\mathbb{L}_2$ be the line passing through the origin and the point $Q := (4, -8, -4)$. Then
$\mathrm{Dist}(\mathbb{L}_2, \mathbb{L}_3)=$ ______ .

___ ___ **g+** Let $\mathsf{G} := \left[\begin{smallmatrix}11 & -18\\6 & -10\end{smallmatrix}\right]$. Compute diagonal matrix

$\mathsf{D}=$ ______ and non-sing $\mathsf{Q}=$ ______
so that $\mathsf{Q}\mathsf{D}\mathsf{Q}^{-1} = \mathsf{G}$. Using #21$^{P.}$312, compute the exponential matrix $\mathrm{e}^{\mathsf{G}} =$ ______ .

___ ___ **h+** Let $\mathsf{M} := \frac{1}{5}\left[\begin{smallmatrix}1 & 2\\4 & 3\end{smallmatrix}\right]$. Compute the limit vector $\lim_{k\to\infty} \mathsf{M}^k \left[\begin{smallmatrix}7\\6\end{smallmatrix}\right]=$ ______ . [*Hint:* M is a Markov matrix.]

i+ On $\mathbb{P} := \mathbb{P}_{<4}$ define an IP $\langle f, g\rangle := \int_{-1}^0 f \cdot g$. Let $\mathcal{E}$ be the ordered-basis $(1, x, x^2, x^3)$. Apply Gram-Schmidt to produce an orthonormal basis

$$\mathcal{B} := (1, h_1, h_2, h_3)$$

for $\mathbb{P}$. Here, each $h_n(x)$ is a polynomial of degree n.
$h_1(x)=$ ______ , $h_2(x)=$ ______ . $h_3(x)=$ ______ .

C2: Prove #16(a,b)$^{P.}$355. (*Jog:* Bessel's Inequality)

C3: #23$^{P.}$324. (*Jog:* Sums of evecs from distinct eigenspaces.)

C4: Matrices $\mathsf{A}, \mathsf{B}, \mathsf{C}, \mathsf{D}$ have sizes $K{\times}K$, $K{\times}L$, $L{\times}K$, $L{\times}L$, resp., forming block matrix $\mathsf{G} := \left[\begin{smallmatrix}\mathsf{A} & \mathsf{B}\\ \mathsf{C} & \mathsf{D}\end{smallmatrix}\right]$, with A invertible. Let $\mathbf{I}$ and $\mathbf{J}$ be the $K{\times}K$ and $L{\times}L$ identity matrices. Prove this block-matrix–formula:

$$\dagger: \quad \begin{bmatrix}\mathbf{I} & \mathbf{0}\\ -\mathsf{C}\mathsf{A}^{-1} & \mathbf{J}\end{bmatrix} \cdot \begin{bmatrix}\mathsf{A} & \mathsf{B}\\ \mathsf{C} & \mathsf{D}\end{bmatrix} = \begin{bmatrix}\mathsf{A} & \mathsf{B}\\ \mathbf{0} & \mathsf{D} - \mathsf{C}\mathsf{A}^{-1}\mathsf{B}\end{bmatrix}.$$

Using that, in (†), the first and third block matrices are block-*triangular*, prove that

$$\ddagger: \quad \mathrm{Det}(\mathsf{G}) = \mathrm{Det}(\mathsf{A}) \cdot \mathrm{Det}(\mathsf{D} - \mathsf{C}\mathsf{A}^{-1} \cdot \mathsf{B}).$$

Suppose now that A and D have the same size (hence $\mathsf{A}, \mathsf{B}, \mathsf{C}, \mathsf{D}$ are each, say, $K{\times}K$). *If* A and C commute, prove that

$$*: \quad \mathrm{Det}(\mathsf{G}) = \mathrm{Det}(\mathsf{A}\mathsf{D} - \mathsf{C}\mathsf{B}).$$

Give a CEX to $(*)$ when $\mathsf{A} \not\rightleftarrows \mathsf{C}$. (Use $K := 2$.)

C1:	___ ___ ___	240pts
C2:	___ ___ ___	120pts
C3:	___ ___ ___	135pts
C4:	___ ___ ___	120pts
Total:	___ ___ ___	615pts

HONOR CODE: *"I have neither requested nor received help on this exam other than from my team-mates and my professor (or his colleague)."* *Name/Signature/Ord*

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