Staple!

Ord: ___ ___

Algebra 1 — MAS4301 3175 — **Home-C** — Prof. JLF King — Thursday 17Apr2003

Hello. The order of your hand-in should be: PROBLEM SHEET (P-S, this side up), TYPESETTING CONVENTIONS (if any), followed by the write-up to the essay. General instructions are on the CHECKLIST. Every "if" should have an explicit "then". Use "$f(x)$ notation" when writing fncs; E.g, write "$\sin(x)$" rather than $\sin x$ or $[\sin x]$.

C1: Please write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed. (Pts: 40×2, 60×2.) **Show NO work!**

___ a In ring $\mathbb{Z}_4[\![x]\!]$, poly $2x+3$ has multiplicative inverse ⌊ . . . . . ⌋.

___ b This deg-4 poly ⌊ . . . . . ⌋ in $\mathbb{Z}_4[\![x]\!]$ is the zero-*function* on $\mathbb{Z}_4$.

___ c In ring $\mathbb{Q}[\![x]\!]$, let **J** be the ideal generated by

$$\begin{aligned}\alpha(x) &:= x^4+4x^3-5x+2\,;\\ \beta(x) &:= x^4+3x^3-3x^2-3x+2\,;\end{aligned}$$

As $\mathbb{Q}[\![x]\!]$ is a PID, there exists a monic intpoly $D(x)=$ ⌊ . . . . . ⌋ st. $\mathbf{J} = D(x)\cdot\mathbb{Q}[\![x]\!]$. [*Hint:* Use the Div.Alg and Eucl.Alg to compute the gcd of $\alpha(x)$ and $\beta(x)$.] Find intpolys st. $\alpha(x) = [$ ⌊ . . . ⌋ $]\cdot D(x)$ and $\beta(x) = [$ ⌊ . . . ⌋ $]\cdot D(x)$.

Your D is in **J**, since $D = \alpha\cdot K + \beta\cdot L$, where $K(x)=$ ⌊ . . . . . ⌋ & $L(x)=$ ⌊ . . . . . ⌋.

___ d Positive integers $A=$ ⌊ . . . ⌋, $B=$ ⌊ . . . ⌋ and $C=$ ⌊ . . . ⌋, each in $[1\,..\,360]$, are such that

$$(x,y,z) \;\mapsto\; [Ax+By+Cz] \mod 360$$

is a ring-iso from $\mathbb{Z}_5\times\mathbb{Z}_8\times\mathbb{Z}_9 \to \mathbb{Z}_{360}$. In $[1\,..\,360]$, integer $n=$ ⌊ . . . . . ⌋ is such that $n\equiv_5 4$ and $n\equiv_8 6$ and $n\equiv_9 3$.

Essay question. Type your solution, in complete, grammatical English sentences that would make Shakespeare weep with admiration. *Double or triple space!* Draw many good, useful Large pictures showing your ideas.

C2: A labeling with posreals, of the cells (squares) of an infinite chessboard, is a ***peg-jump labeling*** if: *For each triple A, B, C of consecutive (vertically or horizontally) cells, both* $\boxed{A \leqslant B+C}$ *and* $\boxed{A+B \geqslant C}$. Tell me the posreal $\beta\in(0,1)$ that you will use to make peg-jump labelings of this form: Each labeling is determined by a integer-valued fnc $N(x,y)$, so that if cell (x,y) is labeled with $\beta^{N(x,y)}$ then the result is peg-jump labeled. Probably, you'll want N to change by at-most-1 in moving from a cell to an adjacent cell.

What to prove. On the infinite chessboard, fill rows $0,-1,-2,\ldots$ with pegs. Give a *careful proof* that it is **impossible** to jump a peg up to row 5. (Pay attention to $\leqslant$ and $<$.) Is it possible to jump to row 4?; if yes, show how.

Now compute, with proof, a posodd $D=2R+1$, so that if the chessboard is completely filled with pegs, with the $D\times D$ cell (centered at the origin) subsequently emptied, then it is impossible to jump a peg to the origin. Extra credit: Generalize either of these to K dimensions. So you'll have an R_K and the "5" will change with K.

End of Home-C

C1:	___ ___ ___	200pts
C2:	___ ___ ___	175pts
Total:	___ ___ ___	375pts

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor (or his colleague)."*
Name/Signature/Ord

Ord: ⌊ . . . . . ⌋