



Due at **BoC, Tues, 02Mar**. Fill-in every blank on this sheet. Write DNE in a blank if the described object does not exist or if the indicated operation cannot be performed. Do **not** approx.: If your result is “ $\sin(\sqrt{\pi})$ ” then write that rather than $.9797\ldots$. Use “ $f(x)$ notation” when writing fncs; in particular, for trig and log fncs. E.g, write “ $\sin(x)$ ” rather than the horrible $\sin x$ or $[\sin x]$.

For the essay questions, TYPESET triple-spaced, grammatical, solutions. A free math-typesetting program is **LAT_EX**; it is customizable to individual preference.

C1: Abbrevs: **parm'ize** for “parametrize”, **parm'tion** for “parametrization”, and **DGPics** for “draw(ing) good pictures”.

For $m \in [1, \infty)$, let Ω_m denote the equi-angular spiral which crosses the x -axis at $Q_1 := (1, 0)$ and, one wrap later, at $Q_m := (m, 0)$. (When $m=1$, the “spiral” degenerates into a circle.) Let $\mathbf{P}_m = (\alpha_m, \beta_m)$ be the **parm'tion** of Ω_m st. $\mathbf{P}_m(0) = Q_1$, $\mathbf{P}_m(2\pi) = Q_m$ and $\mathbf{P}_m(t)$ wraps once whenever t increases by 2π .

So $\alpha_m(t) =$

& $\beta_m(t) =$

a DGPics, computing $L_m := \text{Len}_{\text{wrap}}(Q_1 \curvearrowright Q_m)$ gives $L_m =$

Total-len of Ω_m going in to the origin from Q_1 , is $T_m =$. [Hint: $\int_0^m \sqrt{1 + \beta_m(t)^2} dt$]

As $m \nearrow 1$, geometrically you expect $L_m \rightarrow ??$ and $T_m \rightarrow ??$; do they? (Think l'Hôpital's.) As $m \nearrow \infty$, geometry tells you to expect $T_m \rightarrow ??$, and L_m to be asymptotic to $??$. Are they?

b Fnc $\mathbf{F}(t) := e^t \cdot (\cos(t), \sin(t))$ parametrizes Φ , a **Friendly** spiral. For $\tau \in [-\infty, 0)$, let $\Lambda(\tau)$ be the section of Φ from Q_1 to $\mathbf{F}(\tau)$. **DGPics**, rotate $\Lambda(\tau)$ about the $y = x+3$ line \mathbb{L} , generating a surface-of-revol. whose area is

$\mathcal{A}(\tau) =$

Hence $\mathcal{A}(-\infty) =$

c Let Υ be the part of Φ from $B := \mathbf{F}(-\pi/2)$ to Q_1 . Let R be the region up-from Υ to the x -axis. Let \mathbb{S} be the SoR obtained by rotating R about the x -axis.. **DGPics**, compute the volume of this SoR.

d Showing the interesting steps, compute from $\mathbf{F}()$ the arclength parametrization $\mathbf{A}(s) = (x(s), y(s))$, of the spiral, satisfying that $\mathbf{A}(0) = \mathbf{F}(0)$. Indeed,

$x(s) =$

e Create some *interesting* mathematical problem concerning these spirals. Elegantly solve the problem that you created, drawing nice pictures. *Show off!*

C2: Henceforth, show no work. Simply fill-in each blank on the problem-sheet.

a The quotient and remainder polynomials,

$q(x) =$ _____

& $r(x) =$ _____, _____

satisfy $B = [q \cdot C] + r$ and $\text{Deg}(r) < \text{Deg}(C)$, where $B(x) := 6x^6 + 3x^5 - 6x^4 + 2x^3 + 8x^2 + 1$ and $C(x) := 3x^3 - 3x + 3$.

b Solve prob. #**16P548**, having replaced “70ft” by “80ft”. So the total hydrostatic force on the dam is $\mathbf{\Gamma} =$ _____,

where $\mathbf{\Gamma} :: \frac{\text{lb}}{\text{ft}^3}$ denotes the weight-density of water.

c For $M \in \mathbb{R}$, let \bar{y}_M denote the y -coord of the centroid of R_M , the region in the lying above parabola $y = x^2$ and below line $y = 1 + Mx$, whose 1stquadrant-intersection has x -coord $\mathbf{U} =$ _____.

Fraction $\bar{y}_0 =$ _____ . In terms of M and \mathbf{U} ,

Area(R_M) = _____ and

$$\bar{y}_M = \left[\text{_____} \right] / \text{Area}(R_M).$$

[Hint: The last two blanks are OTForm *Polynomial*(M, \mathbf{U}).]

End of Home-C

C1: _____ 195pts

C2: _____ 95pts

Un- or poorly stapled: _____ -5pts

Total: _____ 290pts

HONOR CODE: *I have neither requested nor received help on this exam other than from my team-mates and my professor (or his colleague).* _____ *Name/Signature/Ord*

Ord: _____

Ord: _____

Ord: _____