



Staple!

ACT
MAA4211 7222

Home-C

Prof. JLF King
Touch: 17Nov2017

Hello. Essays violate the CHECKLIST at *Grade Peril!*
 Exam is due by **3:30PM, Tuesday, 4Nov.2008**, slid completely under my office door, LIT402. Write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

Fill-in all blanks (*handwriting; don't bother to type*).

Definition. For a map $f: \Omega \rightarrow X$ between two MSes, let $\text{Cty}(f)$ denote f 's set of continuity-pts, and use $\text{DisCty}(f)$ for its set of discontinuity-pts. These sets are disjoint and their union is Ω . □

C1: Show no work.

a15 Define $X :=$ $\subset \mathbb{R}$ st. the X -open ball $B := X\text{-Bal}_3(0) =$ satisfies
 $B \subsetneqq \text{Cl}_X(B) =$ $\subsetneqq X\text{-CldBal}_3(0) =$.

b Let $f: \mathbb{R} \rightarrow \mathbb{R}$ by $f(x) := [3x - x^3] - 1$. Define restrictions $g := f|_{[-2,1]}$ and $h := f|_{[-3,3]}$. Then the sup-norm $\|g\|_{\sup} =$ and $\|h\|_{\sup} =$.

c Using the stereographic-metric on \mathbb{R}^+ :
 $\sigma\text{-Diam}(\text{Primes}) =$.

d The limit in the seq **#13^P62** is .

e Define $\mathbf{x} \in \mathbb{R}^{\mathbb{Z}^+}$ by $x_n := 1/3^n$. Each $p \in [1, \infty)$ has $\|\mathbf{x}\|_p =$. And $\|\mathbf{x}\|_p \rightarrow$.

f Let $\varphi: \mathbb{R} \rightarrow \mathbb{R}$ by $\varphi(x) := \begin{cases} 2x & \text{if } x \text{ rational} \\ 1 - 3x & \text{if } x \text{ irrational} \end{cases}$.
 So $\text{Cty}(\varphi) =$ and $\text{Range}(\varphi) =$.

g30 Our MS is $X := \mathbb{R}_+ \times \mathbb{R}_+$ with the usual metric.
 Let $\Omega := \mathbb{Q}_+ \times \mathbb{Q}_+$. Define $f: X \rightarrow \mathbb{R}$ as: For $p \in X \setminus \Omega$, let $f(p) := 0$. For $p = \left(\frac{a}{q}, \frac{b}{r}\right) \in \Omega$, with a, b, q, r posints such that $a \perp q$ and $b \perp r$, let $f(p) := \frac{1}{q} - \frac{2}{r}$. So
 $\text{Cty}(f) =$.

Team: _____

Essay questions: For each question, carefully type a triple-spaced essay solving the problem.

Each essay starts a new page.

C2: A TS X is **countable self-dense (CSD)** if there exists a *countable* subset $D \subset X$ which is X -dense, i.e. $\text{Cl}_X(D) = X$.

i Prove that a seq-cpt MS (X, d) is necessarily CSD.
 You may use, without proof, that a countable union of countable sets is countable.

ii Produce a bounded and CSD MS (Y, m) , together with a seq $\vec{y} \subset Y$ with no Y -convergent subsequence.

C3: Produce (with proof) a sequence \vec{b} of non-negative reals such that $\sum_{n=0}^{\infty} b_n = +\infty$, and such that:

Each monotone subsequence has finite sum.

End of Home-C

C1: _____ 145pts**C2:** _____ 85pts**C3:** _____ 35pts

Poorly stapled, or missing ordinals : _____ -5pts

Missing names, or honor sigs : _____ -5pts

Total: _____ 265pts

HONOR CODE: *"I have neither requested nor received help on this exam other than from my team-mates and my professor (or his colleague)." Name/Signature/Ord*

Ord: _____

Ord: _____

Ord: _____