Staple!

Ord: ___ ___

Sets and Logic
MHF3202 17HE

Class-C

Prof. JLF King
Wednesday, 16Nov2022

C1: Short answer. Show no work. Write LARGE.
Write **DNE** if the object does not exist or the operation cannot be performed. NB: DNE $\neq \{\} \neq 0$.

___ ___ a) The IOP (Individual Optional Project), if you choose to do it, is due by 2PM on Friday, 09Dec2022, slid *completely* under my office door, Little Hall 402 (northeast corner of top floor) Circle: Yes Cool! Thanks

___ ___ b) For a finite list $\mathcal{S}$ of posints, define

$$\boldsymbol{\mu}_{\mathcal{S}}(N) := \Big\{ k \in [1..N] \;\Big|\; \exists d \in \mathcal{S} \text{ with } d \bullet\!| \, k \Big\}.$$

For $\mathcal{S} := \{6, 7, 10\}$, the Inclusion-Exclusion formula (using the floor fnc) for the number of elements, $|\boldsymbol{\mu}_{\mathcal{S}}(N)|$, is

....

(Write your answer, using the floor function as appropriate, in form [term + term + term] − [term + term + term] + term.
The terms are computed from the {6, 7, 10} numbers.)

When $N := 67$, then, $|\mu_{\{6,7,10\}}(67)| =$ ....

___ ___ c) Between sets $\mathbf{X} := \mathbb{Z}_+$ and $\boldsymbol{\Omega} := \mathbb{N}$, consider injections $g\colon \mathbf{X} \hookrightarrow \boldsymbol{\Omega}$ and $h\colon \boldsymbol{\Omega} \hookrightarrow \mathbf{X}$, defined by

$$g(x) := 3x \quad \text{and} \quad h(y) := y + 5\,.$$

Schröder-Bernstein produces a set $B \subset h(\boldsymbol{\Omega}) \subset \mathbf{X}$ st., letting $F := \mathbf{X} \smallsetminus B$, the fnc $\theta\colon \mathbf{X} \hookrightarrow\!\!\!\!\rightarrow \boldsymbol{\Omega}$ is a *bijection*, where

$$*: \qquad \theta|_F := g|_F \quad \text{and} \quad \theta|_B := h^{-1}|_B\,.$$

For this (g, h), the (F, B) pair is unique. Computing, $\theta(\mathbf{56}) =$ .... $\theta(\mathbf{137}) =$ .... $\theta^{-1}(\mathbf{603}) =$ ....

___ ___ d) A "Cantor's-Hotel" type bijection $f\colon (5,6] \hookrightarrow\!\!\!\!\rightarrow (0,1)$ is:
$f($ .... $) :=$ .... , for *each* posint n;
and $f(x) :=$ .... , for *each* $x \in (5,6] \smallsetminus C$,
where $C :=$ ....

___ ___ e) Let $\boldsymbol{\delta}_N$ be the number of derangements of $[1..N]$. Written in Incl-Excl notation (the formula we derived in class),

$\boldsymbol{\delta}_{17} =$ ....

Using binom-coeffs and derangements, the number of N-perms with precisely 3 fixed-points is: ....

[You many use binom-coeffs and $\boldsymbol{\delta}_1, \boldsymbol{\delta}_2, \ldots$ in your answer.]

OYOP: *In grammatical English* **sentences**, *write your essay on every* 2^{nd} *line* (usually), *so I can easily write between the lines.*

C2: [Here, ***WO*** means "Well-ordered".] Consider these sets:

$$B := \{-5 + \tfrac{n}{n+1} \mid n \in \mathbb{N}\}\,;$$
$$C := \{7 - \tfrac{1}{n} \mid n \in \mathbb{Z}_+\}\,;$$
$$D := \{n \cdot \sqrt{2} \mid n \in \mathbb{N}\}\,;$$
$$\mathbb{Q}_{\geq 0} := \Big\{ \frac{p}{q} \;\Big|\; p, q \in \mathbb{N} \text{ with } q \neq 0 \Big\}\,.$$

For each set below, circle one of "**WO**" "**Nope**". In the case of **Nope**, provide an explicit *infinite decreasing sequence* in the given set. [As always, your essay is written in **sentences**.]

B	is	**WO**	**Nope.**
$C \cup D$	is	**WO**	**Nope.**
$\mathbb{Q}_{\geq 0}$	is	**WO**	**Nope.**
$\mathbb{Z} \smallsetminus (-\infty..-17)$	is	**WO**	**Nope.**

C1:	___ ___ ___	150pts
C2:	___ ___	45pts
Total:	___ ___ ___	195pts

NAME: ....

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: ....