Sets and Logic MHF3202 | **SeLo-C** | Prof. JLF King *Touch*: 21Sep2018

C1: *Essay, on your own paper, triple-spaced.*

Prove: THM: *There are ∞ly many primes.*

Start with... PROOF: FTSOContradiction, suppose

$*$: $p_1 < p_2 < \cdots < p_k < \cdots < p_{L-1} < p_L$

is a list of **all** prime numbers. I will now produce a prime q which *differs* from every member of $(*)$, as follows. (*Continue your proof from here.*)

Short answer. For **(C2)** and **(C3)**, show no work; please fill-in each blank on the problem-sheet.

C2: Write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

z Prof. King believes that writing in complete, coherent sentences is crucial in communicating Mathematics, improves posture, and whitens teeth. Circle one:

True! **Yes!** ***wH'at S a?sEnTENcE***

a Repeating decimal $0.1\overline{14}$ equals $\frac{n}{d}$, where posints $n \perp d$ are $n=$ ________ and $d=$ ________.

b Note that $\text{GCD}(15, 21, 35) = 1$. Find particular integers S, T, U so that $15S + 21T + 35U = 1$:

$S=$ ________, $T=$ ________, $U=$ ________.

[*Hint:* $\text{GCD}\big(\text{GCD}(15, 21), 35\big) = 1$.]

c The number of ways of picking 42 objects from 6 types is $\left[\!\left[\begin{matrix}6\\42\end{matrix}\right]\!\right] \overset{\text{Binom}}{\underset{\text{coeff}}{=}} \Big(\qquad\Big)$. And

$\left[\!\left[\begin{smallmatrix}6\\42\end{smallmatrix}\right]\!\right] = \left[\!\left[\begin{smallmatrix}T\\N\end{smallmatrix}\right]\!\right]$, where $T=$ ________ $\neq 6$, and $N=$ ________.

d On $\mathbb{Z}_+$, write $x \$ y$ IFF $\text{GCD}(x, y) \geqslant 2$. So \$ is *Circle*

Transitive: *T F*. **Symm.**: *T F*. **Reflex.**: *T F*.

On $\mathbb{Z}$, say that $x \nabla y$ IFF $x - y < 1$. Then ∇ is:

Trans.: *T F*. **Symm.**: *T F*. **Reflex.**: *T F*.

(Be *careful* on <u>both</u> parts!)

e+ For natnums $J_1, J_2, \ldots, J_K$ and $N := \sum_{\ell=1}^{K} J_\ell$, the ***multinomial coefficient*** $\binom{N}{J_1, J_2, \ldots, J_K}$ *means*

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____.

$\binom{N}{\vec{\mathbf{J}}} \overset{\text{Using}}{\underset{\text{factorials}}{=}}$ ________. $\binom{7}{2,2,3} \overset{\text{Single}}{\underset{\text{integer}}{=}}$ ________.

C3: Posint $N :=$ ________ is such that numbers $N, N{+}1, N{+}2, N{+}3, \ldots, N{+}95$ are each composite. Particular proper divisors in $[2 .. \infty)$ are:

N $\bullet$ ____, $[N{+}1]$ $\bullet$ ____, $[N{+}23]$ $\bullet$ ____, $[N{+}95]$ $\bullet$ ____.