

C1: Show no work. Please write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

Use $\mathcal{V}()$ for the **Vince invariant** of a cell in a TicTacToe board. Use \mathbb{T} to abbreviate "TicTacToe".

a The author of our text is Circle: **Archimedes**
Cantor **DNE** **Euler** **Devlin** **Machen**

b In *Qubic*, $\mathcal{V}(\text{Center cell}) = \dots$. And $\mathcal{V}(\text{Edge cell}) = \dots$. There are \dots many cells whose Vince-invariant equals $\mathcal{V}(\text{Edge cell})$.

c The 7×7 \mathbb{T} board has \dots many TicTacToes. And $4 \times 4 \times 4$ (*Qubic*) has \dots many TicTacToes.

d An explicit bijection $F: \mathbb{N} \leftrightarrow \mathbb{Z}$ is this:
When n is *even*, then $F(n) := \dots$.
When n is *odd*, then $F(n) := \dots$.

e An explicit bijection $g: (-\frac{\pi}{2}, \frac{\pi}{2}) \leftrightarrow \mathbb{R}$ is $g(x) := \dots$.

f **LBolt**: $\text{Gcd}(70, 42) = \dots \cdot 70 + \dots \cdot 42$.
So (**LBolt** again) $G := \text{Gcd}(70, 42, 60) = \dots$ and $\dots \cdot 70 + \dots \cdot 42 + \dots \cdot 60 = G$.

Essay questions: For each question, carefully write a triple-spaced, grammatical, essay solving the problem.

C2: Give an explicit bijection $f: C \rightarrow H$ between intervals $C := (0, 4]$ and $H := (0, 4)$.

C3: **α** The powerset $\mathcal{P}(\Omega)$ of set Ω , is. . . .

β Give a complete proof that there is **no** surjection $h: \Omega \rightarrow \mathcal{P}(\Omega)$. In particular, given a map $h: \Omega \rightarrow \mathcal{P}(\Omega)$, explicitly construct a set $S_h \subset \Omega$ which is *guaranteed* to **not** be in the range of h .

γ When $\Omega := \{M, L, C\}$, the three Stooges, consider this map

$$\begin{aligned} g(M) &:= \{M, L, C\}; \\ g(L) &:= \{M, C\}; \\ g(C) &:= \{\}. \end{aligned}$$

Your $S_g = \{ \dots \}$.

End of Class-C

C1: \dots 150pts

C2: \dots 65pts

C3: \dots 80pts

Total: \dots 295pts

Print
name \dots

Ord: \dots

HONOR CODE: "I have neither requested nor received help on this exam other than from my professor."

Signature: \dots