

**C1:** Show no work. Please write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

Use  $\mathcal{V}()$  for the **Vince invariant** of a cell in a TicTacToe board. Use  $\text{TIT}$  to abbreviate “TicTacToe”.

**a** The author of our text is **Circle**: **Archimedes**  
Cantor DNE Euler Devlin Machen

**b** In *Qubic*,  $\mathcal{V}(\text{Center cell}) = \dots$ . And  
 $\mathcal{V}(\text{Edge cell}) = \dots$ . There are  $\dots$  many cells  
whose Vince-invariant equals  $\mathcal{V}(\text{Edge cell})$ .

**c** The  $7 \times 7$  TIT board has  $\dots$  many TicTacToes. And  
 $4 \times 4 \times 4$  (*Qubic*) has  $\dots$  many TicTacToes.

**d** An explicit bijection  $F: \mathbb{N} \leftrightarrow \mathbb{Z}$  is this:

When  $n$  is *even*, then  $F(n) = \dots$ .

When  $n$  is *odd*, then  $F(n) = \dots$ .

**e** An explicit bijection  $g: (-\frac{\pi}{2}, \frac{\pi}{2}) \leftrightarrow \mathbb{R}$  is  
 $g(x) = \dots$ .

**f** LBolt:  $\text{Gcd}(70, 42) = \dots \cdot 70 + \dots \cdot 42$ .

So (LBolt again)  $G := \text{Gcd}(70, 42, 60) = \dots$  and  
 $\dots \cdot 70 + \dots \cdot 42 + \dots \cdot 60 = G$ .

*Essay questions: For each question, carefully write a triple-spaced, grammatical, essay solving the problem.*

**C2:** Give an explicit bijection  $f: C \rightarrow H$  between intervals  $C := (0, 4]$  and  $H := (0, 4)$ .

**C3:**  **$\alpha$**  The powerset  $\mathcal{P}(\Omega)$  of set  $\Omega$ , is . . .

**$\beta$**  Give a complete proof that there is no surjection  $h: \Omega \rightarrow \mathcal{P}(\Omega)$ . In particular, given a map  $h: \Omega \rightarrow \mathcal{P}(\Omega)$ , explicitly construct a set  $S_h \subset \Omega$  which is guaranteed to not be in the range of  $h$ .



When  $\Omega := \{M, L, C\}$ , the three Stooges, consider this map

$$\begin{aligned} g(M) &:= \{M, L, C\} ; \\ g(L) &:= \{M, C\} ; \\ g(C) &:= \{ \} . \end{aligned}$$

Your  $S_g = \{ \dots \}$ .

End of Class-C

**C1:** \_\_\_\_\_ 150pts

**C2:** \_\_\_\_\_ 65pts

**C3:** \_\_\_\_\_ 80pts

**Total:** \_\_\_\_\_ 295pts

Print name: \_\_\_\_\_ Ord: \_\_\_\_\_

**HONOR CODE:** “I have neither requested nor received help on this exam other than from my professor.”

Signature: \_\_\_\_\_