

C5: Prove that there are infinitely many prime numbers.

End of Exam-C

Note. This is an open brain, open (pristine) Sigmon-Notes exam. Please write solutions for **C3,C4,C5** on separate sheets of paper. Write expressions unambiguously e.g, “ $1/a + b$ ” should be bracketed either $[1/a] + b$ or $1/[a + b]$. (Be careful with **negative** signs!)

Every “if” must be matched by a “then.”

Short answer; show NO work.

C1:

a LBolt gives $G := \text{Gcd}(165, 63) = \underline{\hspace{2cm}}$. And $165S + 63T = G$, where $S = \underline{\hspace{2cm}}$ & $T = \underline{\hspace{2cm}}$ are integers.

b Note that $\text{Gcd}(15, 21, 35) = 1$. Find particular integers S, T, U so that $15S + 21T + 35U = 1$:
 $S = \underline{\hspace{2cm}}$, $T = \underline{\hspace{2cm}}$, $U = \underline{\hspace{2cm}}$.
[Hint: $\text{Gcd}(\text{Gcd}(15, 21), 35) = 1$.]

C2: Let **AT** **AF** **Nei** mean “Always True”, “Always False”, and “Neither”, respectively. Let q represent a general non-zero rational number and let x, y denote general irrationals. Please circle the best answer.

The product $x \cdot y$ is irrational. **AT** **AF** **Nei**
The sum $x + y$ is irrational. **AT** **AF** **Nei**
 $q \cdot x$ is irrational. **AT** **AF** **Nei**
 $q + x$ is irrational. **AT** **AF** **Nei**

Essay questions. Please write in complete sentences.

C3: Prove that $\sqrt{15}$ is irrational.
Now prove that $\sqrt{3} + \sqrt{5}$ is irrational.

C4: Consider two posints A and B . Prove directly, using WellOrd, that there is a posint D st.

$$D\mathbb{Z} = A\mathbb{Z} + B\mathbb{Z}.$$

[Hint: Don’t forget to first show that $A\mathbb{Z} + B\mathbb{Z}$ has a non-zero element.]

C1: _____ 70pts
C2: _____ 40pts
C3: _____ 80pts
C4: _____ 80pts
C5: _____ 80pts

Total: _____ 350pts

Print name _____ Ord: _____

HONOR CODE: “I have neither requested nor received help on this exam other than from my professor.”

Signature: _____

Filename: _____
latex

As of: Monday 31Aug2015. Typeset: 21Feb2017 at 14:07.