Numbers & Polys
MAS3300

Exam-C

Prof. JLF King
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Note. This is an open brain, open (pristine) SigmonNotes exam. Please write each solution on a separate sheet of paper. Write expressions unambiguously e.g, "$1/a+b$" should be bracketed either $[1/a]+b$ or $1/[a+b]$. (Be careful with **negative** signs!)

Each question is an essay question unless specified otherwise.

C1: Show no work. a Let $\Gamma := \text{Gcd}(165, 63)$ and find particular numbers so that $165A + 63B = \Gamma$. Then
$\Gamma =$ ␣ , $A =$ ␣ , $B =$ ␣ .

b Note that $\text{Gcd}(165, 63, 7) = 1$. Find particular numbers C, D, E so that $165C + 63D + 7E = 1$:
$C =$ ␣ , $D =$ ␣ , $E =$ ␣ .
[*Hint:* $\text{Gcd}\big(\text{Gcd}(165, 63), 7\big) = 1$.]

c Let $\mathbb{I}$ comprise the irrational numbers and let $\mathbb{A}$ be the set of algebraic numbers. In terms of cardinality, then, $\mathbb{I}\,?\,\mathbb{A}$ where relation "?" is:

$$\succ \qquad \asymp \qquad \prec$$

(Circle the correct relation.)

C2: Define a "tribonacci" sequence $\vec{\mathbf{b}}$ by: $b_1 := 1$, $b_2 := 2$, $b_3 := 3$ and, for each $n \geqslant 4$,

$$b_n \;:=\; b_{n-1} + b_{n-2} + b_{n-3}\,.$$

Prove, for each $n \geqslant 4$, that $b_n < 2^n$.

C3: Define an open and half-open interval $U := (0, 1)$ and $H := (0, 1]$. Give, with proof, an *explicit* **bijection** $g : U \hookrightarrow H$, by making use of the fact that the subset

$$\left\{\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots\right\}$$

is a "Cantor's Hotel" inside of U. [*Hint:* Your g will necessarily be discontinuous.] For your g, what is $g(\frac{2}{3}) =$ ␣ ?

C4: Here A, B, C, D, E are sets. Let C^D be the set of maps $f : D \to C$. (The exponent is the Domain.) Give, with proof, an *explicit* **bijection**

$$\Phi : A^{D\times E} \to [A^D]^E\,.$$

C5: Carefully write a formal proof that there exist irrational positive reals w, z so that w^z is *rational*, as follows: Let

$$S := \sqrt{7}, \quad T := \sqrt{2} \qquad \text{and} \qquad P := S^T\,.$$

Argue that either S^T is such an example, or P^T is.

Bonus: Create a problem involving induction or number theory or Cantor's theory of Infinities, or rational and algebraic numbers. (Creativity and esthetics count.) Now carefully solve your problem, if you are able in the time you have.

End of Exam-C

C1:	___ ___	70pts
C2:	___ ___	75pts
C3:	___ ___	70pts
C4:	___ ___	70pts
C5:	___ ___	50pts
CBonus:	___ ___	20pts
Total:	___ ___ ___	335pts

Print name ␣ Ord:

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: ␣