Linear Algebra
MAS3114 3241

Class-C

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Hello. Please write **DNE** in a blank *if* the described object does not exist or if the indicated operation cannot be performed.

Do ***not*** approx.: If your result is "$\sin(\sqrt{\pi})$" then write that rather than .9797···.

C1: Show no work.

a Suppose C and A are 3×3 matrices such that $\mathrm{Det}(C) = \frac{1}{2}$ and $\mathrm{Det}(A) = 3$. Then

$\mathrm{Det}\big(C^{-1}AC^{t}A^{t}AC^{t}\big)=$ ______________.

b $M := \begin{bmatrix} -7 & 9 & 0 \\ -6 & 8 & 0 \\ 12 & -18 & -1 \end{bmatrix}$ has three real eigenvalues,

$\alpha=$ ________ $\leqslant \beta=$ ________ $\leqslant \gamma=$ ________.

Hence $\begin{bmatrix} \alpha & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & \gamma \end{bmatrix} = U^{-1}MU$, where

$U = \Bigg[\quad \Bigg| \quad \Bigg| \quad \Bigg]$.

c $\mu=$ ________ $\leqslant \nu =$ ________

are the eigenvalues of $G := \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$. Let $D := \begin{bmatrix} \mu & 0 \\ 0 & \nu \end{bmatrix}$. Then $D = U^{-1}GU$ where the 2×2 *integer* matrix U is

$U = \Bigg[\quad \Bigg]$.

d Find 2×2 matrices E and G with $E^2 - G^2$ **unequal** to $[E-G]\cdot[E+G]$: $E :=$ ________. $G :=$ ________.

$E^2 - G^2=$ ________. $[E-G][E+G]=$ ________.

e Let $\mathbf{u} := (1+2i, 3-i, 1)$ and $\mathbf{w} := (1, 1+i, 2-3i)$. Then

$\|\mathbf{u}\|=$ ________ and $\langle \mathbf{u}, \mathbf{w} \rangle=$ ________. Thus

$\mathbf{p} := \mathrm{Proj}_{\mathbf{u}}(\mathbf{w})=$ ______________ and

$\mathbf{r} := \mathrm{Orth}_{\mathbf{u}}(\mathbf{w})=$ ______________.

[*Hint:* $\mathrm{Proj}_{\mathbf{u}}(\mathbf{p})$ should equal $\mathbf{p}$. DYCheck $\mathbf{r} \perp \mathbf{p}$?]

f Matrix $M := \frac{1}{5}\left[\begin{smallmatrix} 1 & 2 \\ 4 & 3 \end{smallmatrix}\right]$ is a Markov matrix. Its unique stationary (column) vector is $\mathbf{v}= \Big[\quad \Big]$.

Essay question: On your own sheets of paper, write a soln using complete sentences.

C2: Let $B := \begin{bmatrix} 1 & 2 & 1 & 0 & 1 \\ 3 & 6 & 0 & -3 & 0 \end{bmatrix}$. Then $R := \mathit{RREF}(B)$ is [show no work, here]

$R = \Big[\quad \Big| \quad \Big| \quad \Big| \quad \Big| \quad \Big]$.

I For subspace $\mathbf{V} := \mathrm{Nul}(B)$, use back-substitution, and *scaling*, to produce an *integer basis*

$\mathbf{v}_1:=($, , , , $)$ $\mathbf{v}_2:=($, , , , $)$

$\mathbf{v}_3:=($, , , , $)$. [Show No Work, here!]

[*Note:* Only use as many as the dimension of $\mathbf{V}$.]

II Using *sentences* and *pictures*, explain & show the Gram-Schmidt algorithm computing an **orthogonal** *integer-basis* for $\mathbf{V}$:

$\mathbf{b}_1:=($, , , , $)$, $\mathbf{b}_2:=($, , , , $)$

$\mathbf{b}_3:=($, , , , $)$. [Entries are integers]

Arrange that the Gcd of the entries in each vector is 1, and that the first non-zero value is positive. (Do *not* show computation of inner-products.)

C1: ___ ___ ___ 180pts

C2: ___ ___ ___ 110pts

Total: ___ ___ ___ 290pts

Print name ______________ Ord:

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: ______________