C1: SHORT ANSWER. Fill in the blanks below, expressing your answer in simplest form unless otherwise indicated. Write **DNE**, for "Does Not Exist", if the indicated operation cannot be performed.

Do not make approximations. **Show no work.** There is no partial credit for this question, so carefully verify that you have written what *you* mean. In particular, make sure that you write expressions unambiguously, e.g the expression "$1/a + b$" should be parenthesized either $(1/a) + b$ or $1/(a + b)$ so that I know your meaning. Be careful with negative signs. [Points $30 * 10 = 300$]

$$A := \begin{bmatrix} & & 1 & 1 & 2 \\ & 2 & 5 & 1 & \\ & 1 & 2 & & -1 \\ 1 & & 1 & & \end{bmatrix}, \quad B := \begin{bmatrix} -1 & -1 \\ 2 & -4 \end{bmatrix}, \quad C := \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}.$$

(a1) $\operatorname{rank}(A) =$ ____________ $\operatorname{nullity}(A) =$ ____________.

(a2) Give a basis for $\operatorname{Null}(A)$. (Write the vectors vertically.) Basis= $\{ \quad \}$

(a3) Give a basis for $\operatorname{Row}(A)$. (Write the transpose of each vector.) Basis= $\{ \quad \}$

(a4) Give a basis for $\operatorname{Col}(A)$. Basis= $\{ \quad \}$

(a5) What value of y puts $\begin{bmatrix} 1 \\ y \\ 3 \\ 3 \end{bmatrix}$ in $\operatorname{Col}(A)$? $y =$ ______.

(b1) Compute the char. poly $\operatorname{cp}_B(x) =$ __________________, and set of eigen-values: $\{$ ______ $\}$.

(b2) Let α denote the more-positive eigenvalue, and β the more-negative. Compute a B-eigenvector, $\mathbf{v}_\alpha$, for α; compute a B-eigenvector, $\mathbf{v}_\beta$, for β: $\mathbf{v}_\alpha =$ ______, $\mathbf{v}_\beta =$ ______.

(c) Let $p()$ be the smallest degree polynomial such that $p(-1) = 1 = p(1)$ and $p(3) = 13$. Then $p(x) =$ ____________________

(d) Write $\det(C)$ as a degree-3 polynomial in the 9 variables $a, b, \ldots, h, i$. $\det(C) =$ ________________________________

(e) $E(y)$ is a 10×10 matrix populated (Thanks Scott!) with numbers, except that $E_{5,4} = E_{5,9} = y$. I tell you that $\det\big(E(1)\big) = 1$ and $\det\big(E(2)\big) = 3$. $\det\big(E(3)\big) =$ __________.

C2: [70 Points] This continues from the matrix B from problem C1(b). Please show the steps for this problem (and also fill-in the blanks):

Just as we did in class, compute a 2×2 change-of-basis matrix M so that $M^{-1}\begin{bmatrix}\alpha & 0\\ 0 & \beta\end{bmatrix}M = B$.

$M =$ ....

Let z denote the entry in position (2,1) of matrix B^{100}. Write z as a linear combination of α^{100} and β^{100} (using the actual values of α and β, of course.)

$z =$ ....

Bonus [15 Points] Make up a nice Linear Algebra problem –something creative. You do not necessarily need to know how to solve it.

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